C H A P T E R 12

ATOMS

12.1 THOMSON'S MODEL OF AN ATOM

1. *Describe Thomson's model of an atom. Why was this model discarded later on ?*

Thomson's model of an atom. In 1898, *J.J. Thomson* proposed that an atom is a sphere of positively charged matter with electrons embedded in it. The positive charge is uniformly distributed over the entire atom. The arrangement of electrons inside the continuous positive charge is similar to that of the *seeds in a watermelon or the plums in a pudding.* That is why Thomson's atomic model is also known as ***plum pudding model***. The electrons are arranged in such a manner that their mutual repulsions are balanced by their attractions with the positively charged matter. Thus the atom as a whole is stable and neutral.

Thomson's model was able to explain with some success the processes like chemical reaction and radioactive disintegration. To explain the observed spectra of elements, Thomson assumed that slight perturbations of atoms cause vibrations of the electrons about their equilibrium positions. These vibrating electrons emit electromagnetic radiations of their own frequency of oscillations.

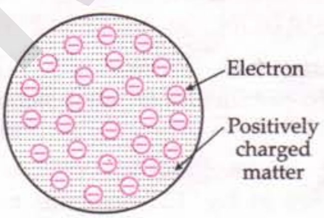

Fig. 12.1 Plum pudding model of an atom.

Failure of Thomson's model. Thomson model remained popular till about 1911 and was discarded later on due to the following drawbacks :

1. It could not explain the origin of several spectral series in the case of hydrogen and other atoms.
2. It failed to explain the large angle scattering of α-particles in Rutherford's experiment.

12.2 ALPHA PARTICLE SCATTERING EXPERIMENT

2. *Describe Rutherford's experiment on the scattering of α-particles by a nucleus. Explain the observations and conclusions of the experiment.*

Scattering of α-particles : Geiger-Marsden experiment. On the suggestion of *Rutherford*, his two associates *H. Geiger* and *E. Marsden*, in 1911, performed experiments on the scattering of *α-particles* from thin foils and obtained an important insight into the structure of the atom.

An α-particle is a helium ion, i.e., a helium atom from which both the electrons have been removed. It has charge equal to + 2*e* and its mass is nearly four times the mass of a proton.

Experimental arrangement. A schematic arrangement of the Geiger-Marsden experiment is shown in Fig. 12.2. A radioactive source of α-particles like $^{214}_{83}Bi$ is enclosed in thick lead block, provided with a narrow opening. The α-particles from this source are collimated into a narrow beam through a narrow slit. The beam is allowed to fall on a thin gold foil of thickness 2.1×10^{-7} m. The α-particles scattered in different directions are observed with the help of a rotatable detector which consists of a zinc sulphide screen and a microscope. Whenever an α-particle strikes the screen, it produces a tiny flash, or scintillation, which is viewed through the microscope. In this way, the number of α-particles scattered at different angles can be counted. The whole apparatus is enclosed in an evacuated chamber to avoid scattering of α-particles by air molecules.

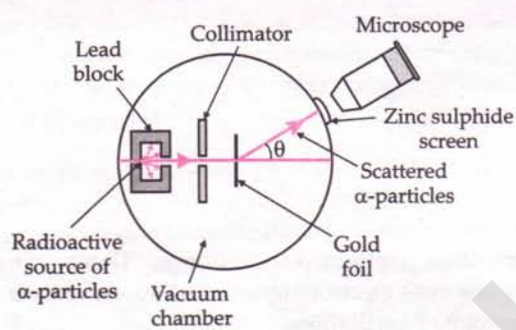

Fig. 12.2 α-particle scattering experiment.

Observations. As shown in Fig. 12.3, a graph is drawn between the scattering angle θ and the number $N(\theta)$ of the α-particles scattered at an angle θ, for a very large number of α-particles.

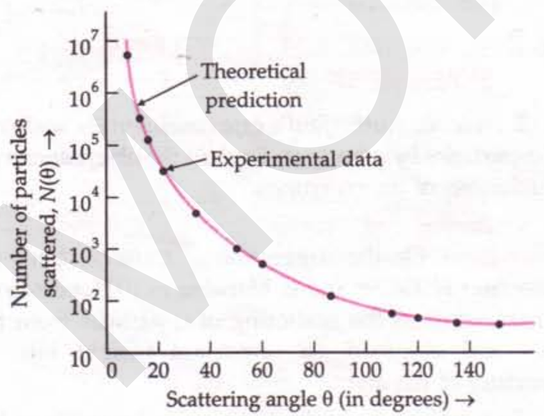

Fig. 12.3 Graph of the total number of α-particles scattered at different angles θ.

The above graph reveals the following facts :

1. Most of the α-particles pass straight through the gold foil or suffer only small deflections.
2. A few α-particles, about 1 in 8,000, get deflected through 90° or more.
3. Occasionally, an α-particle gets rebounded from the gold foil, suffering a deflection of nearly 180°.

Significance of the result. Rutherford concluded the following important facts about an atom :

1. As most of the α-particles pass straight through the foil, so most of the space within atoms must be empty.
2. To explain large angle scattering of α-particles, Rutherford suggested that all the positive charge and the mass of the atom is concentrated in a very small region, called the nucleus of the atom.
3. The nucleus is surrounded by a cloud of electrons whose total negative charge is equal to the total positive charge on the nucleus so that the atom as a whole is electrically neutral.

The scattering of the α-particles is due to the Coulombic repulsion between the positively charged α-particles and the nuclei. An α-particle like 1 or 1′, passing through the atom at large distance from the nucleus, experiences small repulsion and passes almost undeflected. But the α-particles like $(2, 2')$, $(3, 3')$ which pass closer to the nucleus, experience large repulsive forces and hence scatter through large angles. Very rarely, an α-particle like 4 travels head-on towards the nucleus. The strong repulsive force slows down the α-particle, which is finally stopped and then repelled back along its original path.

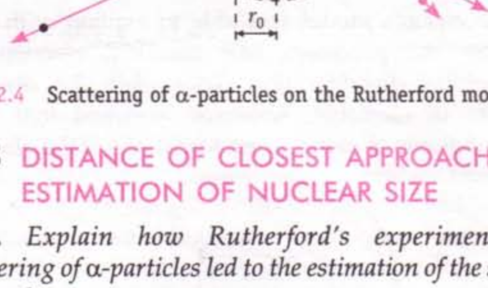

Fig. 12.4 Scattering of α-particles on the Rutherford model.

12.3 DISTANCE OF CLOSEST APPROACH : ESTIMATION OF NUCLEAR SIZE

3. *Explain how Rutherford's experiment on scattering of α-particles led to the estimation of the size of the nucleus.*

Distance of closest approach : Estimation of nuclear size. As shown in Fig. 12.5, suppose an α-particle of mass m and initial velocity v moves directly towards the centre of the nucleus of an atom. As it approaches the positive nucleus, it experiences Coulombic repulsion and its kinetic energy gets progressively converted into electrical energy. At a certain distance r_0 from the nucleus, the α-particle stops for a moment and then begins to retrace its path, *i.e.*, it is scattered through an angle of 180°. The distance r_0 is called the ***distance of closest approach***. At this distance r_0, the entire initial kinetic energy of the α-particle gets converted into electrostatic potential energy.

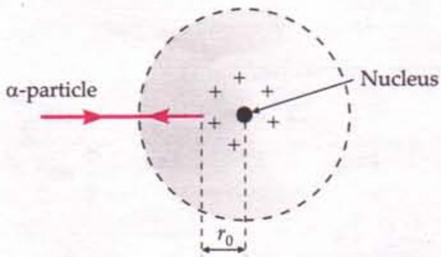

Fig. 12.5 Distance of closest approach.

Now, charge on an α-particle, $q_1 = +2e$

Charge on a scattering nucleus, $q_2 = +Ze$

where Z is the atomic number of foil atoms.

Initial kinetic energy of α-particle, $K_\alpha = \frac{1}{2} mv^2$

Electrostatic P.E. of α-particle and nucleus at distance r_0,

$$U = k \cdot \frac{q_1 q_2}{r_0} = k \cdot \frac{2e \cdot Ze}{r_0}$$

By conservation of energy, $K_\alpha = U$

or $$K_\alpha = \frac{1}{2} mv^2 = k \cdot \frac{2Ze^2}{r_0}$$

$$\therefore \quad r_0 = \frac{2kZe^2}{K_\alpha} = \frac{4k\, Ze^2}{mv^2}$$

where $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{ C}^{-2}$.

Clearly, the radius of the nucleus must be smaller than r_0.

In one of the Rutherford's experiments, α-particles of energy 5.5 MeV were used.

$$\therefore \quad K_\alpha = 5.5 \text{ MeV} = 5.5 \times 1.6 \times 10^{-13} \text{ J}$$

Atomic number of gold, $Z = 79$

$$\therefore \quad r_0 = \frac{2k\, Ze^2}{K_\alpha}$$

$$= \frac{2 \times 9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{5.5 \times 1.6 \times 10^{-13}}$$

$$= 4.13 \times 10^{-14} \text{m} = 41.3 \text{ fm}$$

This distance is considerably larger than the sum of the radii of the gold nucleus and the α-particle. *The radius of a nucleus is of the order of a fermi, where 1 fermi (fm)* $= 10^{-15}$ *m*.

12.4 IMPACT PARAMETER

4. *What is meant by impact parameter ? How does it influence the shape of the trajectory of an α-particle in its scattering from a heavy nucleus ? What is the value of impact parameter for a head-on collision ?*

Impact parameter. The scattering of an α-particle from a nucleus depends on its distance of closest approach to the nucleus or on an equivalent length, called the **impact parameter** '*b*' as shown in Fig. 12.6.

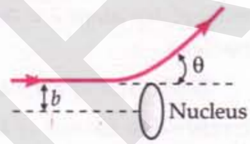

Fig. 12.6 Impact parameter *b* and scattering angle θ.

The impact parameter is defined as the perpendicular distance of the velocity vector of the α-particle from the centre of the nucleus, when it is far away from the atom.

From experiments, one can notice the following points :

1. For large impact parameters, the repulsive force experienced by the α-particle is weak (because of its inverse square law character) and the α-particle passes almost undeflected.
2. For small impact parameter, the repulsive force is large and so the α-particle is scattered through large angle.
3. *For a head-on collision*, when the α-particle just aims at the centre of the nucleus, *the impact parameter* $b = 0$, *scattering angle* $\theta = 180°$, *i.e.*, the α-particle is reversed back along its original path.

Thus the shape of the trajectory of the scattered α-particles depends on the impact parameter and the nature of the potential field.

Rutherford deduced the following relationship between the impact parameter *b* and the scattering angle θ :

$$b = \frac{1}{4\pi\varepsilon_0} \frac{Ze^2 \cot\frac{\theta}{2}}{K}$$

or $$b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot\frac{\theta}{2}}{\frac{1}{2} mv^2}$$

The scattering experiments provide a method for investigating the nature of the forces involved.

12.5 RUTHERFORD'S MODEL OF AN ATOM AND ITS LIMITATIONS

5. *Explain the Rutherford's model of an atom. What are its limitations ?*

Rutherford's model of an atom. On the basis of the α-particle scattering experiment, Rutherford proposed the following model of an atom :

1. *An atom consists of a small and massive central core in which the entire positive charge and almost the whole mass of the atom are concentrated. This core is called the* ***nucleus***.

2. *The size of the nucleus ($\approx 10^{-15}$ m) is very small as compared to the size of the atom ($\approx 10^{-10}$ m).*

3. *The nucleus is surrounded by a suitable number of electrons so that their total negative charge is equal to the total positive charge on the nucleus and the atom as a whole is electrically neutral.*

4. *The electrons revolve around the nucleus in various orbits just as planets revolve around the sun. The centripetal force required for their revolution is provided by the electrostatic attraction between the electrons and the nucleus.*

Limitations of Rutherford's atomic model :

1. According to *electromagnetic theory,* an accelerated charged particle must radiate electromagnetic energy. An electron revolving around the nucleus is under continuous acceleration towards the centre. It should continuously lose energy and move in orbits of gradually decreasing radii. The electron should follow a spiral path and finally it should collapse into the nucleus. Thus the ***Rutherford's model cannot explain the stability of an atom.***

2. In Rutherford's model, an electron can revolve in orbits of all possible radii. So it should emit a continuous spectrum. But an atom like hydrogen always emits a discrete line spectrum.

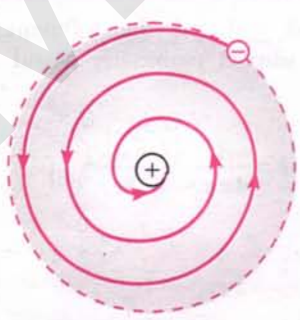

Fig. 12.7 Spiral path of an accelerated electron.

Examples based on Distance of Closest Approach and Impact Parameter

Formulae Used

1. K.E. of α-particle, $K = \frac{1}{2} mv^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{r_0}$

2. Distance of closest approach,

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{K} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Ze^2}{mv^2}$$

3. Impact parameter,

$$b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot\frac{\theta}{2}}{K} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot\frac{\theta}{2}}{\frac{1}{2} mv^2}$$

Units Used

Distances r_0 and b are in metre, kinetic energy K in joule, velocity v in ms^{-1}.

Constants Used

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}, \quad e = 1.6 \times 10^{-19} \text{ C}.$$

Example 1. *What is the distance of closest approach when a 5.0 MeV proton approaches a gold nucleus ?*

Solution. At the distance r_0 of closest approach,

K.E. of a proton

= P.E. of proton and the gold nucleus

$$K = \frac{1}{2} mv^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze \cdot e}{r_0} \quad [q_1 = Ze, q_2 = e]$$

or $$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{K}$$

But $K = 5.0 \text{ MeV} = 5.0 \times 1.6 \times 10^{-13}$ J

For gold, $Z = 79$

$$\therefore \quad r_0 = \frac{9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{5.0 \times 1.6 \times 10^{-13}}$$

$$= 2.28 \times 10^{-14} \text{ m} \simeq \mathbf{2.3 \times 10^{-14} \text{ m}}.$$

Example 2. *In a Geiger-Marsden experiment, what is the distance of closest approach to the nucleus of a 7.7 MeV α-particle before it comes momentarily to rest and reverses its direction ?* [NCERT ; CBSE D 15C]

Solution. Here $K = 7.7 \text{ MeV} = 7.7 \times 1.6 \times 10^{-13}$ J, $Z(\text{Au}) = 79$

At the distance of closed approach of an α-particle,

$$K = \frac{1}{2} mv^2 = \frac{1}{4\pi\varepsilon_0} \frac{Ze \cdot 2e}{r_0} \quad [q_1 = Ze, q_2 = 2e]$$

$$\therefore \quad r_0 = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{K}$$

$$= \frac{9 \times 10^9 \times 2 \times 79 \times (1.6 \times 10^{-19})^2}{7.7 \times 1.6 \times 10^{-13}} \text{ m}$$

$$= 29.5 \times 10^{-15} \text{ m} \approx \mathbf{30 \text{ fm}} \qquad [1 \text{ fm} = 10^{-15} \text{ m}]$$

The radius of gold nucleus is less than 30 fm. Infact, the actual radius of gold nucleus is 6 fm.

Example 3. *A beam of α-particles of velocity $2.1 \times 10^7 \; ms^{-1}$ is scattered by a gold foil ($Z = 79$). Find the distance of closest approach of the α-particle to the gold nucleus. The value of charge/mass for α-particle is $4.8 \times 10^7 \; C \, kg^{-1}$.*

Solution. At the distance of closest approach,

Electrostatic P.E. = Initial K.E. of α-particle

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze \cdot 2e}{r_0} = \frac{1}{2} mv^2$$

$$\therefore \quad r_0 = \frac{2}{4\pi\varepsilon_0} \cdot \frac{Ze}{v^2} \cdot \frac{2e}{m}$$

$$= 2 \times 9 \times 10^9 \times \frac{79 \times 1.6 \times 10^{-19}}{(2.1 \times 10^7)^2} \times 4.8 \times 10^7$$

$$= \mathbf{2.5 \times 10^{-14} \text{ m.}}$$

Example 4. *An α-particle after passing through a potential difference of 2×10^6 V falls on a silver foil. The atomic number of silver is 47. Calculate (i) the kinetic energy of the α-particle at the time of falling on the foil (ii) the kinetic energy of the α-particle at a distance of 5×10^{-14} m from the nucleus and (iii) the shortest distance from the nucleus of silver to which the α-particle reaches.*

Solution. (*i*) Charge on α-particle, $q = 2e$,

$$V = 2 \times 10^6 \text{ V}$$

K.E. of α-particle,

$$K = qV = 2 \times 1.6 \times 10^{-19} \times 2 \times 10^6 = 6.4 \times 10^{-13} \text{ J}$$

(*ii*) Charge on silver nucleus = $Ze = 47\,e$

P.E. of the α-particle at a distance of 5×10^{-14} m from the silver nucleus

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{47e \times 2e}{5 \times 10^{-14}}$$

$$= \frac{9 \times 10^9 \times 94 \times (1.6 \times 10^{-19})^2}{5 \times 10^{-14}} = 4.3 \times 10^{-13} \text{ J}$$

So, 4.3×10^{-3} J of K.E. gets converted into P.E.

∴ K.E. of the α-particle at a distance of 5×10^{-14} m from the silver nucleus

$$= 6.4 \times 10^{-13} - 4.3 \times 10^{-13}$$

$$= \mathbf{2.1 \times 10^{-13} \text{ J.}}$$

(*iii*) Distance of closest approach,

$$r_0 = \frac{2k\,Ze^2}{K} = \frac{2 \times 9 \times 10^9 \times 47 \times (1.6 \times 10^{-10})^2}{6.4 \times 10^{-13}}$$

$$= \mathbf{3.4 \times 10^{-14} \text{ m.}}$$

Example 5. *The number of particles scattered at 60° is 100 per minute in an α-particle scattering experiment, using gold foil. Calculate the number of particles per minute scattered at 90° angle.*

Solution. Number of α-particles scattered at an angle θ,

$$N \propto \frac{1}{\sin^4(\theta/2)} \quad i.e., \quad N = \frac{K}{\sin^4(\theta/2)},$$

where K is a proportionality constant

$$\therefore \quad \frac{N_{90°}}{N_{60°}} = \frac{\sin^4(60°/2)}{\sin^4(90°/2)}$$

or

$$N_{90°} = \frac{\sin^4 30°}{\sin^4 45°} \times N_{60°} = \left[\frac{1/2}{1/\sqrt{2}}\right]^4 \times 100$$

$$= \frac{100}{4} = \mathbf{25 \text{ particles min}^{-1}.}$$

Example 6. *Calculate the impact parameter of a 5 MeV particle scattered by 90° when it approaches a gold nucleus.*

Solution. Here $K = 5 \text{ MeV} = 5 \times 1.6 \times 10^{-13}$ J,

$\theta = 90°, \quad Z = 79$

Impact parameter,

$$b = \frac{kZe^2 \cot\frac{\theta}{2}}{K}$$

$$= \frac{9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2 \cot 45°}{5 \times 1.6 \times 10^{-13}}$$

$$= \mathbf{2.27 \times 10^{-14} \text{ m.}}$$

Problems For Practice

1. In a head-on collision between an α-particle and a gold nucleus, the distance of closest approach is 41.3 fermi. Calculate the energy of the particle. (1 fermi = 10^{-15} m). (**Ans.** 5.51 MeV)
2. An α-particle of kinetic energy 10^{-12} joule exhibits back scattering from a gold nucleus ($Z = 79$). What can be the maximum possible radius of the gold nucleus ? (**Ans.** 3.64×10^{-14} m)
3. In a head-on collision between an α-particle and a gold nucleus, the minimum distance of approach is 3.95×10^{-14} m. Calculate the energy of the α-particle. (**Ans.** 6 MeV)

4. An α-particle of kinetic energy 7.68 MeV is projected towards the nucleus of copper ($Z = 29$). Calculate its distance of nearest approach. (**Ans.** 1.09×10^{-14} m)

5. An α-particle of energy 4 MeV is scattered by gold foil ($Z = 79$). Calculate the maximum volume in which positive charge of the atom is likely to be concentrated. (**Ans.** $7.7 \times 10^{-40} \text{m}^3$)

6. In a Geiger-Marsden experiment, calculate the distance of closest approach to the nucleus of $Z = 80$, when an α-particle of 8 MeV energy impinges on it before it comes momentarily to rest and reverses its direction. **[CBSE OD 12]** (**Ans.** 28.2 fm)

7. What is the impact parameter at which the scattering angle is 10° for $Z = 79$ and initial energy of the α-particle is 5 MeV ? (**Ans.** 2.6×10^{-15} m)

HINTS

1. Here $r_0 = 41.3 \text{ fm} = 41.3 \times 10^{-15} \text{m}$, $Z = 79$

$$K = \frac{1}{4\pi \varepsilon_0} \cdot \frac{2Ze^2}{r_0}$$

$$= \frac{9 \times 10^9 \times 2 \times 79 \times (1.6 \times 10^{-19})^2}{41.3 \times 10^{-15}} \text{ J}$$

$$= 8.814 \times 10^{-13} \text{J} = \frac{8.814 \times 10^{-13}}{1.6 \times 10^{-13}} \text{MeV} = \mathbf{5.51 \text{ MeV}}.$$

2. $$r_0 = \frac{2k\,Ze^2}{K} = \frac{2 \times 9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{10^{-12}}$$

$$= \mathbf{3.64 \times 10^{-14} \text{ m}}.$$

3. $$K = \frac{2k\,Ze^2}{r_0} = \frac{2 \times 9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{3.95 \times 10^{-14} \times 1.6 \times 10^{-13}} \text{ MeV}$$

$$= \mathbf{6 \text{ MeV}}.$$

4. $$r_0 = \frac{2k\,Ze^2}{K} = \frac{2 \times 10^9 \times 29 \times (1.6 \times 10^{-19})^2}{7.68 \times 1.6 \times 10^{-13}} \text{ m}$$

$$= \mathbf{1.09 \times 10^{-14} \text{ m}}.$$

5. $$r_0 = \frac{2kZe^2}{K} = \frac{2 \times 9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{4 \times 1.6 \times 10^{-13}}$$

$$= 5.688 \times 10^{-14} \text{m}$$

Required volume, $V_0 = \frac{4}{3} \pi r_0^3 = \mathbf{7.7 \times 10^{-40} \text{ m.}}$

6. Here $Z = 80$, $K = 8 \text{ MeV} = 8 \times 1.6 \times 10^{-13} \text{ J}$

$$r_0 = \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{K}$$

$$= \frac{9 \times 10^9 \times 2 \times 80 \times (1.6 \times 10^{-19})^2}{8 \times 1.6 \times 10^{-13}} \text{ m}$$

$$= 28.2 \times 10^{-15} \text{ m} = \mathbf{28.2 \text{ fm}}$$

7. $$b = \frac{kZe^2 \cot \theta / 2}{K}$$

$$= \frac{9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2 \cot 5°}{5 \times 1.6 \times 10^{-13}}$$

$$= \frac{9 \times 79 \times 1.6 \times 10^{-16}}{5 \times 0.0875} \qquad [\tan 5° = 0.0875]$$

$$= \mathbf{2.6 \times 10^{-13} \text{ m.}}$$

12.6 BOHR'S QUANTISATION CONDITION

6. *Obtain Bohr's quantisation condition on the basis of the wave picture of an electron.*

Bohr's quantisation condition of angular momentum. Consider the motion of an electron in a circular orbit of radius r around the nucleus of the atom. According to de Broglie hypothesis, this electron is also associated with wave character. Hence a circular orbit can be taken to be a stationary energy state only if it contains an integral number of de-Broglie wavelengths, *i.e.*, we must have

$$2\pi r = n\lambda$$

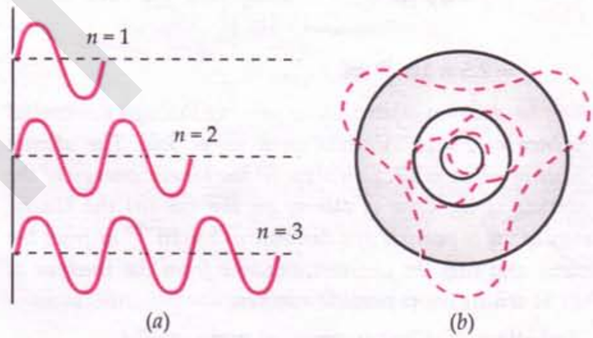

Fig. 12.8 de Broglie waves and hydrogen atom
(*a*) Only a certain number of (de Broglie) wavelengths would fit in the electron orbits, and
(*b*) discrete standing waves (characteristic frequencies or wavelengths) correspond to the discrete orbits in Bohr's theory.

But de Broglie wavelength, $\lambda = \frac{h}{mv}$

$$\therefore \qquad 2\pi r = \frac{nh}{mv}$$

The angular momentum L of the electron must be

$$L = mvr = \frac{nh}{2\pi}, \qquad n = 1, 2, 3.....$$

This is the famous ***Bohr's quantisation condition for angular momentum***. Thus only those circular orbits can be the allowed stationary states of an electron in which its angular momentum is an integral multiple of $h/2\pi$.

12.7 BOHR'S ATOMIC MODEL : POSTULATES

7. *State the postulates of Bohr's theory of hydrogen atom.*

Postulates of Bohr's theory of hydrogen atom. Accepting the Rutherford's nucleus model of an atom as well as the Planck's quantum theory, Bohr proposed an atomic model to explain the spectra emitted by hydrogen atoms. Bohr's atomic model, so called *Planetary model of the atom,* is based on the following postulates :

1. ***Nuclear concept.*** *An atom consists of a small and massive central core, called nucleus around which planetary electrons revolve. The centripetal force required for their rotation is provided by the electrostatic attraction between the electrons and the nucleus.*

2. ***Quantum condition.*** *Of all the possible circular orbits allowed by the classical theory, the electrons are permitted to circulate only in those orbits in which the angular momentum of an electron is an integral multiple of* $\frac{h}{2\pi}$; *h being Planck's constant.* Therefore, for any permitted orbit,

$$L = mvr = \frac{nh}{2\pi}, \quad n = 1, 2, 3......$$

where L, m and v are the angular momentum, mass and speed of the electron, r is the radius of the permitted orbit and n is positive integer called *principal quantum number.* The above equation is *Bohr's famous quantum condition.*

3. ***Stationary orbits.*** *While revolving in the permissible orbits, an electron does not radiate energy. These non-radiating orbits are called stationary orbits.*

4. ***Frequency condition.*** *An atom can emit or absorb radiation in the form of discrete energy photons only when an electron jumps from a higher to a lower orbit or from a lower to a higher orbit, respectively.* If E_1 and E_2 are the energies associated with these permitted orbits, then the frequency ν of the emitted or absorbed radiation is given by

$$h\nu = E_2 - E_1$$

This is Bohr's famous frequency condition.

12.8 BOHR'S THEORY OF HYDROGEN ATOM

8. *Using Bohr's postulates, derive an expression for the radii of the permitted orbits in the hydrogen atom. Show that the speed of electron in the innermost orbit of H-atom is 1/137 times the speed of light. Also obtain an expression for the total energy of an electron in the nth orbit of an atom. What does negative value of this energy signify ? What is Bohr's radius ?*

Bohr's Theory of hydrogen atom : Radii of permitted orbits. According to Bohr's theory, a hydrogen atom consists of a nucleus with a positive charge Ze, and a single electron of charge $-e$, which revolves around it in a circular orbit of radius r. Here Z is the atomic number and for hydrogen $Z = 1$. The electrostatic force of attraction between the nucleus and the electron is

$$F = \frac{k\,Ze.e}{r^2} = \frac{k\,Ze^2}{r^2}$$

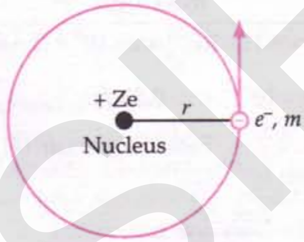

Fig. 12.9 Bohr's model of hydrogen atom.

To keep the electron in its orbit, the centripetal force on the electron must be equal to the electrostatic attraction. Therefore,

$$\frac{mv^2}{r} = \frac{k\,Ze^2}{r^2}$$

or $$mv^2 = \frac{k\,Ze^2}{r} \quad ...(1)$$

or $$r = \frac{k\,Ze^2}{mv^2} \quad ...(2)$$

where m is the mass of the electron and v, its speed in an orbit of radius r.

Bohr's *quantisation condition for angular momentum is*

$$L = mvr = \frac{nh}{2\pi}$$

or $$r = \frac{nh}{2\pi mv} \quad ...(3)$$

From equations (2) and (3), we get

$$\frac{k\,Ze^2}{mv^2} = \frac{nh}{2\pi mv}$$

or $$v = \frac{2\pi k\,Ze^2}{nh} \quad ...(4)$$

Substituting this value of v in equation (3), we get

$$r = \frac{nh}{2\pi m} \cdot \frac{nh}{2\pi k\,Ze^2}$$

or $$r = \frac{n^2 h^2}{4\pi^2 mk\,Ze^2} \quad ...(5)$$

Clearly, the radii of the permitted orbits are proportional to n^2 and increase in the ratio of 1 : 4 : 9 : 16.... The parameter n is called the *principle quantum number*.

The radius of the innermost orbit of the hydrogen atom, called ***Bohr's radius*** can be determined by putting $Z = 1$ and $n = 1$ in equation (5) and it is denoted by r_0.

$$\therefore \quad r_0 = \frac{h^2}{4\pi^2 \, mke^2}$$

$$= \frac{(6.63 \times 10^{-34})^2}{4\pi^2 \times 9.11 \times 10^{-31} \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2} \text{ m}$$

$$= 5.29 \times 10^{-11} \text{ m} \simeq \mathbf{0.53 \text{ Å}}.$$

Speed of electron. From equation (4), the orbital speed of electron is

$$v = \frac{2\pi k Ze^2}{nh}$$

For hydrogen, $Z = 1$, therefore,

$$v = \frac{2\pi ke^2}{nh} = \left(\frac{2\pi ke^2}{ch}\right)\frac{c}{n}$$

or $$v = \alpha \,. \frac{c}{n}$$

The quantity $\alpha = \frac{2\pi ke^2}{ch}$, is a dimensionless constant and is called ***fine structure constant***. Its value is

$$\alpha = \frac{2\pi \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2}{3 \times 10^8 \times 6.63 \times 10^{-34}} = \frac{1}{137}$$

$$\therefore \quad v = \frac{1}{137} \,. \frac{c}{n}$$

For first orbit ($n = 1$), $v = \frac{c}{137}$

Thus the speed of the electron in the innermost orbit is 1/137 of the speed of light in vacuum, while the speed in the outer orbits varies inversely with n.

Energy of the electron. It includes the electron's kinetic energy and the electrostatic potential energy of the two charges.

Kinetic energy of the electron in nth orbit is

$$\text{K.E.} = \frac{1}{2} mv^2 = \frac{k\, Ze^2}{2r} \quad \text{[Using equation (1)]}$$

Potential energy of the electron in nth orbit is

$$\text{P.E.} = k\, \frac{q_1 q_2}{r} = \frac{k\, Ze.(-e)}{r} = -\frac{k\, Ze^2}{r}$$

Hence total energy of the electron in nth orbit is

$$E_n = \text{K.E.} + \text{P.E.}$$

$$= \frac{k\, Ze^2}{2r} - \frac{k\, Ze^2}{r} = -\frac{k\, Ze^2}{2r}$$

$$= -\frac{k\, Ze^2}{2} \cdot \frac{4\pi^2\, mk\, Ze^2}{n^2 h^2}$$

[Using equation (5)]

or $$E_n = -\frac{2\pi^2\, mk^2\, Z^2 e^4}{n^2 h^2} \quad \text{...(6)}$$

The negative value of the total energy indicates that the electron is bound to the nucleus by means of electrostatic attraction and some work is required to be done to pull it away from the nucleus.

12.9 SPECTRAL SERIES OF HYDROGEN ATOM

9. *On the basis of Bohr's theory, explain the origin of the various spectral series of hydrogen atom.*

Spectral series of hydrogen atom. From Bohr's theory, the energy of an electron in the nth orbit of hydrogen atom is given by

$$E_n = -\frac{2\pi^2\, mk^2\, Z^2\, e^4}{h^2} \cdot \frac{1}{n^2}$$

According to *Bohr's frequency condition,* whenever an electron makes a transition from a higher energy level n_2 to a lower energy level n_1, the difference of

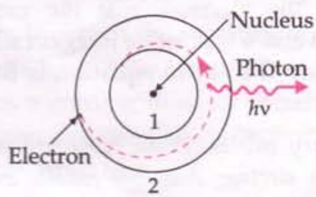

Fig. 12.10 Emission of a spectral line by a hydrogen atom.

energy appears in the form of a photon. The frequency ν of the emitted photon is given by

$$h\nu = E_{n_2} - E_{n_1}$$

or $$h\nu = \frac{2\pi^2\, mk^2 e^4}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$\nu = \frac{2\pi^2\, mk^2 e^4}{h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

As $c = \nu\lambda$, therefore **wave number** $\bar{\nu}$, *which is the reciprocal of wavelength* λ, is given by

$$\bar{\nu} = \frac{1}{\lambda} = \frac{\nu}{c} = \frac{2\pi^2\, mk^2 e^4}{ch^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $$\bar{\nu} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

where $R = \dfrac{2\pi^2 mk^2 e^4}{ch^3}$, is the *Rydberg constant* and its value is $1.0973 \times 10^7\ \text{m}^{-1}$. The above equation is the **Rydberg formula** for the spectrum of hydrogen atom. This formula indicates that the radiation emitted by the excited hydrogen atom consists of certain specific wavelengths or frequencies, the values of which depend on quantum numbers n_1 and n_2.

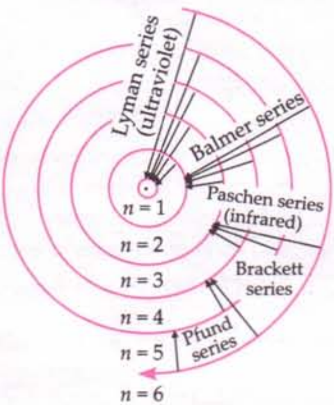

Fig. 12.11 Spectral series of hydrogen atom.

As shown in Fig. 12.11, the origin of the various series in the hydrogen spectrum can be explained as follows :

(*i*) ***Lyman series.*** If an electron jumps from any higher energy level $n_2 = 2, 3, 4......$ to a lower energy level $n_1 = 1$, we get a set of spectral lines called *Lyman series* which belong to the *ultraviolet region* of the electromagnetic spectrum. This series is given by

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right], \quad n_2 = 2, 3, 4......$$

(*ii*) ***Balmer series.*** The spectral series corresponding to the transitions $n_2 = 3, 4, 5......$ to $n_1 = 2$, lies in the *visible region* and is called *Balmer series*. For this series,

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right], \quad n_2 = 3, 4, 5......$$

(*iii*) ***Paschen series.*** If $n_2 = 4, 5, 6......$ and $n_1 = 3$, we get a spectral series in the *infrared region* which is called *Paschen series.*

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right], \quad n_2 = 4, 5, 6......$$

(*iv*) ***Brackett series.*** If $n_2 = 5, 6, 7......$ and $n_1 = 4$, we get a spectral series in the *infrared region* which is called *Brackett series.*

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right], \quad n_2 = 5, 6, 7......$$

(*v*) ***Pfund series.*** If $n_2 = 6, 7, 8......$ and $n_1 = 5$, we get a spectral series in the *infrared region* which is called *Pfund series.*

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right], \quad n_2 = 6, 7, 8......$$

The greatness of Bohr's theory lies in the fact that it not only successfully explained the already known series of Lyman, Balmer and Paschen but also predicted two new series in the infrared region which were later on discovered by *Brackett* (in 1922) and *Pfund* in (1924). *Neil Bohr* was awarded the 1922 Nobel prize in physics for this work.

12.10 ENERGY LEVEL DIAGRAM FOR HYDROGEN

10. *What is the energy level diagram for an atom ? Calculate the energies of the various energy levels of a hydrogen atom and draw an energy level diagram for it.*

Energy level diagram. *It is a diagram in which the energies of the different stationary states of an atom are represented by parallel horizontal lines, drawn according to some suitable energy scale.* Such a diagram illustrates more clearly the known facts about the stationary states and the emission or absorption of various spectral lines.

Energy levels of hydrogen. From Bohr's theory, the total energy of an electron in nth orbit is given by

$$E_n = -\frac{2\pi^2 mk^2 Z^2 e^4}{n^2 h^2}$$

For hydrogen $Z = 1$, therefore,

$$E_n = -\frac{2\pi^2 mk^2 e^4}{h^2} \cdot \frac{1}{n^2}$$

Energy of the electron in the first orbit $(n = 1)$ *is given by*

$$E_1 = -\frac{2 \times (3.14)^2 \times 9.11 \times 10^{-31} \times (9 \times 10^9)^2 \times (1.6 \times 10^{-19})^4}{(6.63 \times 10^{-34})^2} \cdot \frac{1}{1^2}$$

$$= -21.76 \times 10^{-19}\ \text{J} = -\frac{21.76 \times 10^{-19}}{1.6 \times 10^{-19}}\ \text{eV}$$

$$= \mathbf{-13.6\ eV} \qquad [\because 1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}]$$

Hence we can write,

$$E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2}\ \text{eV}$$

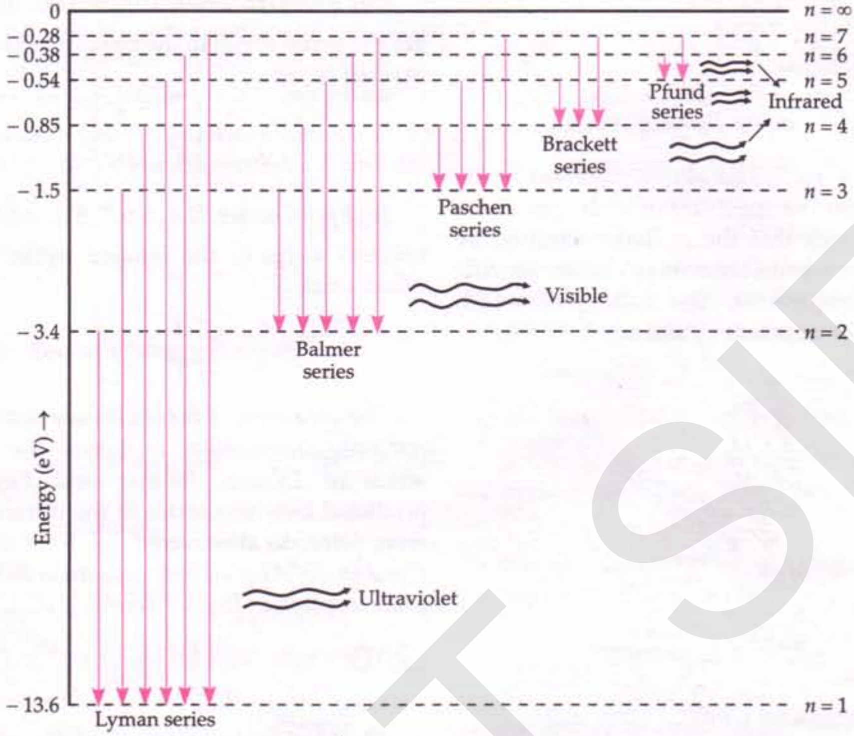

Fig. 12.12 Energy level diagram of hydrogen atom and its various spectral series.

Setting $n = 2, 3, 4$, etc., we get the energies for the higher orbits as follows :

$$E_2 = \frac{-13.6}{2^2} = -\mathbf{3.4\ eV}$$

$$E_3 = \frac{-13.6}{3^2} = -\mathbf{1.51\ eV}$$

$$E_4 = \frac{-13.6}{4^2} = -\mathbf{0.85\ eV}$$

$$E_5 = \frac{-13.6}{5^2} = -\mathbf{0.54\ eV}$$

$$E_6 = \frac{-13.6}{6^2} = -\mathbf{0.38\ eV}$$

Clearly, *an electron can have only certain definite values of energy while revolving in different orbits*. This is called ***energy quantisation***.

The energy state corresponding to $n = 1$, has the lowest energy equal to −13.6 eV. *This state or orbit is called the **ground or normal state*** of the atom. Normally the electron in the hydrogen atom remains in the ground state. When the hydrogen atom receives energy from outside by processes such as electron collisions, the electron may make transition to some higher energy state like E_2, E_3, E_4, etc. which are called the ***excited states.***

Figure 12.12 shows the energy level diagram of the hydrogen atom in which the energies of the various permitted orbits have been represented by parallel horizontal lines according to some energy scale. The principal quantum number n labels the energy states in the ascending order of energy.

12.11 LIMITATIONS OF BOHR'S THEORY

11. *State the limitations of Bohr's theory of hydrogen atom.*

Limitations of Bohr's theory. Although Bohr's theory could successfully explain the spectrum of hydrogen, yet it had the following shortcomings :

1. This theory is applicable only to hydrogen-like single electron atoms and fails in the case of atoms with two or more electrons.
2. In the spectrum of hydrogen, certain spectral lines are not single lines but a group of closed lines with slightly different frequencies. Bohr's theory could not explain these fine features of the hydrogen spectrum.
3. It does not explain why only circular orbits should be chosen when elliptical orbits are also possible.

4. As electrons exhibit wave properties also, so orbits of electrons cannot be exactly defined as in Bohr's theory.
5. Bohr's theory does not tell anything about the relative intensities of the various spectral lines. Bohr's theory predicts only the frequencies of these lines.
6. It does not explain the further splitting of spectral lines in a magnetic field **(*Zeeman effect*)** or in an electric field **(*Stark effect*)**.

12.12 EXCITATION AND IONISATION POTENTIALS

12. *Define the terms excitation and ionisation energies, and excitation and ionisation potentials.*

Excitation energy. *The excitation energy of an atom is defined as the energy required by its electron to jump from the ground state to any one of the excited states.*

First excitation energy of hydrogen

$$= E_2 - E_1 = -3.4 - (-13.6) = \mathbf{10.2\ eV}$$

Second excitation energy of hydrogen

$$= E_3 - E_1 = -1.51 - (-13.6) = \mathbf{12.09\ eV.}$$

Ionisation Energy. *It is defined as the energy required to knock an electron completely out of the atom, i.e.,* the energy required to take an electron from its ground state to the outermost orbit ($n = \infty$). After the removal of the electron, the atom is left with positive charge and it is said to *ionised*.

Ionisation energy of hydrogen

$$= E_\infty - E_1 = 0 - (-13.6) = \mathbf{13.6\ eV.}$$

Excitation potential. *It is that accelerating potential which gives to a bombarding electron, sufficient energy to excite the target atom by raising one of its electrons from an inner to an outer orbit.*

First excitation potential of hydrogen

$$= -3.4 - (-13.6) = \mathbf{10.2\ V}$$

Second excitation potential of hydrogen

$$= -1.51 - (-13.6) = \mathbf{12.09\ V.}$$

Ionisation potential. *It is that accelerating potential which gives to a bombarding electron, sufficient energy to ionise the target atom by knocking one of its electrons completely out of the atom.*

Ionisation potential of hydrogen

$$= 0 - (-13.6) = \mathbf{13.6\ V.}$$

Examples based on Bohr's Theory of Hydrogen Atom

Formulae Used

1. $\frac{kZe^2}{r^2} = \frac{mv^2}{r}$
2. $L = mvr = \frac{nh}{2\pi}$
3. $h\nu = E_{n_2} - E_{n_1}$
4. $r_n = \frac{n^2h^2}{4\pi^2 mkZe^2}$
5. $v_n = \frac{2\pi ke^2}{nh} = \alpha \cdot \frac{c}{n}$

   where $\alpha = \frac{2\pi k e^2}{ch}$, is fine structure constant
6. $\text{K.E.} = \frac{kZe^2}{2r}$
7. $\text{P.E.} = \frac{-kZe^2}{r}$
8. Total energy,

   $E_n = -\frac{2\pi^2 mk^2 Z^2 e^4}{n^2h^2} = -\frac{Rhc}{n^2} = \frac{-13.6}{n^2}\text{ eV}$
9. $E_n = -\text{K.E.}$ or $\text{K.E.} = -E_n$; $\text{P.E.} = -2\,\text{K.E.} = 2E_n$
10. Frequency, $\nu = \frac{2\pi^2 mk^2 e^4}{h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$
11. Wave number, $\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

    where $R = \frac{2\pi^2 mk^2 e^4}{ch^3}$, is Rydberg's constant
12. Ionisation potential $= -\frac{13.6\,Z^2}{n^2}$ volt
13. $T_n = \frac{2\pi r_n}{v_n} = \frac{n^3h^3}{4\pi^2 mk^2 Ze^4} = T_1 n^3$

Units Used

Energy E_n is in joule, r and λ are in metre, wave number $\bar{\nu}$ in m^{-1}.

Constants Used

$R = 1.097 \times 10^7\ m^{-1}$, $h = 6.63 \times 10^{-34}$ Js,

$m_e = 9.1 \times 10^{-31}$ kg

Example 7. *Evaluate Rydberg's constant by putting the values of the fundamental constants in its expression.*

Solution. Rydberg's constant is defined by the expression

$$R = \frac{2\pi^2 mk^2 e^4}{ch^3}$$

$$= \frac{2 \times 9.87 \times 9.1 \times 10^{-31} \times (9 \times 10^9)^2 \times (1.6 \times 10^{-19})^4}{3 \times 10^8 \times (6.63 \times 10^{-34})^3}\ m^{-1}$$

$$= \mathbf{1.0973 \times 10^7\ m^{-1}.}$$

Example 8. *Calculate the value of 'fine structure constant'.*

Solution. Fine structure constant is defined as

$$\alpha = \frac{2\pi ke^2}{ch}$$

$$= \frac{2 \times 3.14 \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2}{3 \times 10^8 \times 6.63 \times 10^{-34}}$$

$$= \frac{\mathbf{1}}{\mathbf{137}} = \mathbf{0.0073.}$$

Example 9. *What is the angular momentum of an electron in the third orbit of an atom ?* [CBSE F 93]

Solution. Here $n = 3, \quad h = 6.6 \times 10^{-34}$ Js

Angular momentum,

$$L = \frac{nh}{2\pi} = \frac{3 \times 6.6 \times 10^{-34} \times 7}{2 \times 22}$$

$$= \mathbf{3.15 \times 10^{-34}\ Js.}$$

Example 10. *Write down the expression for the radii of orbits of hydrogen atom. Calculate the radius of the smallest orbit.* [CBSE OD 94]

Solution. The radius of the *n*th orbit of a hydrogen atom is given by

$$r = \frac{n^2h^2}{4\pi^2 mke^2}$$

Radius of the innermost orbit, called *Bohr's radius*, is obtained by putting $n = 1$. It is denoted by r_0.

$$\therefore \quad r_0 = \frac{h^2}{4\pi^2 mke^2}$$

$$= \frac{(6.6 \times 10^{-34})^2}{4\pi^2 \times 9.1 \times 10^{-31} \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2} \text{ m}$$

$$\simeq 0.53 \times 10^{-10} \text{ m} = \mathbf{0.53\ Å.}$$

Example 11. *Calculate the velocity of electron in Bohr's first orbit of hydrogen atom. How many times does the electron go in Bohr's first orbit in one second ?*

Solution. The velocity of electron in Bohr's *n*th orbit is

$$v = \frac{2\pi k e^2}{nh}$$

$\therefore$ Velocity of electron in Bohr's first ($n = 1$) orbit is

$$v = \frac{2\pi ke^2}{h}$$

$$= \frac{2 \times 3.14 \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.63 \times 10^{-34}}$$

$$= \mathbf{2.186 \times 10^6\ ms^{-1}.}$$

Frequency of revolution of electron,

$$f = \frac{1}{T} = \frac{1}{2\pi r / v} = \frac{v}{2\pi r}$$

$$= \frac{2.186 \times 10^6}{2 \times 3.14 \times 0.53 \times 10^{-10}} = \mathbf{6.57 \times 10^{15}\ Hz.}$$

[From example 10, $r = 0.53 \times 10^{-10}$ m]

Example 12. *Determine the speed of the electron in $n = 3$ orbit of He^+ ion.* [NCERT]

Solution. Here $n = 3, \quad Z = 2$

$$v = \frac{2\pi kZe^2}{nh}$$

$$= \frac{2 \times 3.14 \times 9 \times 10^9 \times 2 \times (1.6 \times 10^{-19})^2}{3 \times 6.6 \times 10^{-34}}$$

$$= \mathbf{1.46 \times 10^6\ ms^{-1}.}$$

Example 13. *Show that the speed of an electron in the innermost orbit of H-atom is 1/137 times the speed of light in vacuum.*

Solution. Speed of an electron in the innermost ($n = 1$) orbit of H-atom is

$$v = \frac{2\pi ke^2}{h} = \frac{2\pi ke^2}{ch} \cdot c$$

$$= \frac{2\pi \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2}{3 \times 10^8 \times 6.63 \times 10^{-34}} \cdot c = \frac{\mathbf{1}}{\mathbf{137}}\ \mathbf{c.}$$

Example 14. *Calculate the period of revolution of an electron revolving in the first orbit of hydrogen atom. Given radius of first orbit = 0.53 Å and $c = 3 \times 10^8\ ms^{-1}$.*

Solution. Velocity of electron in *n*th orbit $= \frac{1}{137} \cdot \frac{c}{n}$

Period of revolution of an electron in first orbit,

$$T = \frac{2\pi r}{v} = \frac{2\pi r \times 137 \times n}{c}$$

$$= \frac{2 \times 3.14 \times 0.53 \times 10^{-10} \times 137 \times 1}{3 \times 10^8} \text{ s}$$

$$= \mathbf{1.52 \times 10^{-16}\ s.} \qquad [\because\ n = 1]$$

Example 15. *The energy of an electron in the nth orbit is given by $E_n = -13.6 / n^2$ eV. Calculate the energy required to excite an electron from ground state to the second excited state.* [CBSE Sample Paper 98]

Solution. Energy in ground state ($n = 1$),

$$E_1 = -\frac{13.6}{1^2} = -13.6 \text{ eV}$$

Energy in second excited state ($n = 3$),

$$E_3 = -\frac{13.6}{3^2} = -1.51 \text{ eV}$$

Required energy $= E_3 - E_1$

$$= -1.51 - (-13.6) = \mathbf{12.09\ eV.}$$

Example 16. *It is found experimentally that 13.6 eV energy is required to separate a hydrogen atom into a proton and an electron. Compute the orbital radius and the velocity of the electron in a hydrogen atom.* [NCERT]

Solution. Total energy of the electron in hydrogen atom is

$$E = -13.6 \text{ eV} = -13.6 \times 1.6 \times 10^{-19} \text{ J}$$

As $E = -\dfrac{ke^2}{r}$

$$\therefore \quad r = -\frac{ke^2}{E}$$

$$= +\frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{13.6 \times 1.6 \times 10^{-19}} = \mathbf{5.3 \times 10^{-11} \text{ m.}}$$

$$\text{K.E.} = \frac{1}{2} mv^2 = -E = 13.6 \times 1.6 \times 10^{-19} \text{ J}$$

$$\therefore \quad v = \sqrt{\frac{2 \times 13.6 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = \mathbf{2.2 \times 10^6 \text{ ms}^{-1}}.$$

Example 17. *According to the classical electromagnetic theory, calculate the initial frequency of the light emitted by the electron revolving around a proton in hydrogen atom.* [NCERT]

Solution. In the above example, we have seen that velocity of electron moving around a proton in hydrogen atom in an orbit of radius 5.3×10^{-11} m is 2.2×10^6 m/s. Thus the frequency of revolution of the electron revolving around the proton is

$$f = \frac{v}{2\pi r} = \frac{2.2 \times 10^6 \text{ ms}^{-1}}{2 \times \left(\frac{22}{7}\right) \times (5.3 \times 10^{-11} \text{ m})}$$

$$= \mathbf{6.6 \times 10^{15} \text{ Hz}}$$

As the frequency of the electromagnetic wave emitted by the revolving electron is equal to its frequency of revolution, so the initial frequency of the light emitted is 6.6×10^{15} Hz.

Example 18. *A 10 kg satellite circles earth once every 2 h in an orbit having a radius of 8000 km. Assuming that Bohr's angular momentum postulate applies to satellites just as it does to an electron in the hydrogen atom, find the quantum number of the orbit of the satellite.* [NCERT]

Solution. Here $m = 10$ kg, $T = 2$ h $= 7200$ s

$$r = 8000 \text{ km} = 8 \times 10^6 \text{ m}$$

According to the Bohr's quantisation condition of angular momentum,

$$L = mvr = \frac{nh}{2\pi} \quad \text{or} \quad m \times \frac{2\pi r}{T} \times r = \frac{nh}{2\pi}$$

or $$n = \frac{4\pi^2 r^2 m}{Th}$$

$$= \frac{4 \times 9.87 \times (8 \times 10^6)^2 \times 10}{7200 \times 6.63 \times 10^{-34}} = \mathbf{5.3 \times 10^{45}}.$$

Example 19. *In the ground state of hydrogen atom, its Bohr radius is given as 5.3×10^{-11} m. The atom is excited such that the radius becomes 21.2×10^{-11} m. Find (i) the value of the principal quantum number and (ii) the total energy of the atom in this excited state.* [CBSE D13 C]

Solution. (*i*) As $r \propto n^2$

$$\therefore \quad \left(\frac{n_2}{n_1}\right)^2 = \frac{r_2}{r_1} = \frac{21.2 \times 10^{-11}}{5.3 \times 10^{-11}} = 4$$

or $$\frac{n_2}{n_1} = 2$$

or $$n_2 = 2n_1 = 2 \times 1 = \mathbf{2.}$$

(*ii*) $$E_2 = \frac{E_1}{2^2}$$

$$= \frac{-13.6}{4} = \mathbf{-3.4 \text{ eV}.}$$

Example 20. *The ground state energy of hydrogen atom is −13.6 eV.*

(*i*) *What is the kinetic energy of an electron in the 2nd excited state ?*

(*ii*) *What is the potential energy of an electron in the 3rd excited state ?*

(*iii*) *If the electron jumps to the ground state from the 3rd excited state, calculate the wavelength of the photon emitted.* [CBSE OD 08]

Solution. Here $E_1 = -13.6$ eV

$$E_3 = \frac{E_1}{3^2} = \frac{-13.6}{9} = -1.51 \text{ eV}$$

$$E_4 = \frac{E_1}{4^2} = \frac{-13.6}{16} = -0.85 \text{ eV}$$

(*i*) K.E. of an electron in 2nd excited state

$$= -E_3 = \mathbf{1.51 \text{ eV}.}$$

(*ii*) P.E. of an electron in 3rd excited state

$$= 2E_4 = \mathbf{-1.70 \text{ eV}.}$$

(*iii*) $$E_4 - E_1 = -0.85 - (-13.6)$$

$$= 12.75 \text{ eV} = 12.75 \times 1.6 \times 10^{-19} \text{ J}$$

As $$E_4 - E_1 = h\nu = \frac{hc}{\lambda}$$

$$\therefore \quad \lambda = \frac{hc}{E_4 - E_1} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{12.75 \times 1.6 \times 10^{-19}} \text{ m}$$

$$= \mathbf{970 \text{ Å}.}$$

Example 21. *The energy level diagram of an element is given in Fig. 12.13. Identify, by doing necessary calculations, which transition corresponds to the emission of a spectral line of wavelength 102.7 nm.* [CBSE D 08]

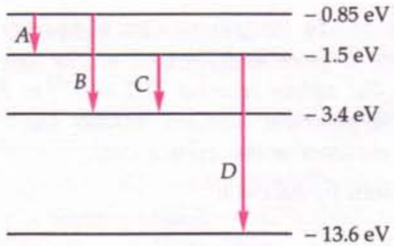

Fig. 12.13

Solution. Here $\lambda = 102.7\ \text{nm} = 102.7 \times 10^{-9}\text{m}$

The energy of the emitted photon is

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{102.7 \times 10^{-9}}$$

$$= 1.9355 \times 10^{-18}\text{J}$$

$$= \frac{1.9355 \times 10^{-18}}{1.6 \times 10^{-19}}\ \text{eV} = 12.097\ \text{eV}$$

$$\approx 12.1\ \text{eV}$$

This energy corresponds to the transition D for which the energy change $= -1.5 - (-13.6) = \mathbf{12.1\ eV}$.

Example 22. *The energy of the electron, in the hydrogen atom, is known to be expressible in the form*

$$E_n = -\frac{13.6\ eV}{n^2} \quad n = 1, 2, 3, ...$$

Use this expression to show that the

(*i*) *electron in the hydrogen atom cannot have an energy of –6.8 eV.*

(*ii*) *spacing between the lines (consecutive energy levels) with in the given set of the observed hydrogen spectrum decreases as n increases.* [CBSE OD 08C]

Solution. Given $E_n = -\frac{13.6\ \text{eV}}{n^2}$

Putting $n = 1, 2, 3, ...$, we get

$$E_1 = -\frac{13.6}{1^2} = -13.6\ \text{eV}\ ;$$

$$E_2 = -\frac{13.6}{2^2} = -3.4\ \text{eV}$$

$$E_3 = -\frac{13.6}{3^2} = -1.51\ \text{eV}\ ;$$

$$E_4 = -\frac{13.6}{4^2} = -0.85\ \text{V}$$

$$E_\infty = -\frac{13.6}{\infty^2} = 0\ \text{eV}$$

(*i*) Clearly, an electron in the hydrogen atom cannot have an energy of – 6.8 eV

(*ii*) As the value of n increases, the energy difference between two consecutive energy levels decreases.

Example 23. *Figure 12.14 shows energy level diagram of hydrogen atom.*

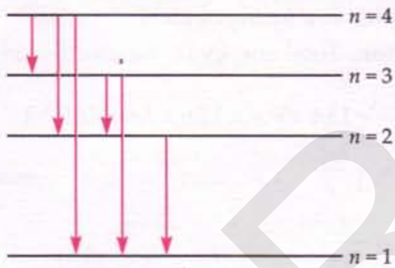

Fig. 12.14

(*i*) *Find out the transition which results in the emission of a photon of wavelength 496 nm.*

(*ii*) *Which transition corresponds to the emission of radiation of maximum wavelength ? Justify your answer.* [CBSE OD 15C]

Solution. (*i*) The energy levels of a hydrogen atom are :

$$E_1 = -13.6\ \text{eV}, \quad E_2 = -3.4\ \text{eV},$$
$$E_3 = -1.51\ \text{eV}, \quad E_4 = -0.85\ \text{eV}$$

Also, $h = 6.63 \times 10^{-34}\ \text{Js}$, $c = 3 \times 10^{8}\ \text{ms}^{-1}$

Energy of a photon of wavelength 496 nm,

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{496 \times 10^{-9} \times 1.6 \times 10^{-19}}\ \text{eV}$$

$$\simeq 2.5\ \text{eV}$$

Now, $E_4 - E_2 = -0.85 + 3.4 = 2.55 \simeq E$

Hence, the transition from $n = 4$ to $n = 2$ level results in the emission of a photon of wavelength 496 nm.

(*ii*) The transition $n = 4$ to $n = 3$ level corresponds to the emission of maximum wavelength because this transition gives out a photon of minimum energy.

Example 24. *Using the Rydberg formula, calculate the wavelengths of the first four spectral lines in the Lyman series of the hydrogen spectrum.* [NCERT]

Solution. The wavelengths of various spectral lines in the Balmer series are obtained by putting $n_f = 1$ and $n_i = 2, 3, 4, ...$ in the Rydberg formula :

$$\bar{v} = \frac{1}{\lambda_{\text{if}}} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] = R\left[\frac{1}{1^2} - \frac{1}{n_i^2}\right]$$

or $$\lambda_{\text{if}} = \frac{n_i^2}{R(n_i^{\,2} - 1)} = \frac{n_i^2}{1.097 \times 10^{7}\ \text{m}^{-1}\,(n_i^2 - 1)}$$

$$= \frac{0.912 \times 10^{-7} n_i^2}{n_i^2 - 1}$$

Various spectral lines in Lyman series are

$$\lambda_{21} = \frac{0.912 \times 10^{-7} \times 4}{3} \text{ m} = \mathbf{1216\ Å}$$

$$\lambda_{31} = \frac{0.912 \times 10^{-7} \times 9}{8} \text{ m} = \mathbf{1026\ Å}$$

$$\lambda_{41} = \frac{0.912 \times 10^{-7} \times 16}{15} \text{ m} = \mathbf{972.8\ Å}$$

$$\lambda_{51} = \frac{0.912 \times 10^{-7} \times 25}{24} \text{ m} = \mathbf{950\ Å}$$

Example 25. *Using the Rydberg formula, calculate the wavelength of the first four spectral lines in the Balmer series of the hydrogen spectrum.*

Solution. The wavelengths of various spectral lines in the Balmer series can be obtained by putting $n_1 = 2$ and $n_2 = 3, 4, 5, ...$ in the Rydberg formula :

$$\bar{\nu} = \frac{1}{\lambda_{if}} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right],$$

where $R = 1.097 \times 10^7 \text{m}^{-1}$.

∴ Various spectral lines in Balmer series are

$$\lambda_{32} = \frac{1}{1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9}\right)} \text{ m}$$

$$= 6563 \times 10^{-10} \text{m} = \mathbf{6563\ Å}$$

$$\lambda_{42} = \frac{1}{1097 \times 10^7 \left(\frac{1}{4} - \frac{1}{16}\right)} \text{ m}$$

$$= 4862 \times 10^{-10} \text{m} = \mathbf{4862\ Å}$$

$$\lambda_{52} = \frac{1}{1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{25}\right)} \text{ m}$$

$$= 4341 \times 10^{-10} \text{ m} = \mathbf{4341\ Å}$$

$$\lambda_{62} = \frac{1}{1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{36}\right)} \text{ m}$$

$$= 4102 \times 10^{-10} \text{ m} = \mathbf{4102\ Å}$$

Example 26. *Calculate the shortest wavelength of Lyman series. Given Rydberg's constant, $R = 10967700\ m^{-1}$.*

Solution. Lyman series is given by the wave number,

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$$

For shortest wavelength of Lyman series, $n = \infty$. It is given by

$$\frac{1}{\lambda_s} = R\left[\frac{1}{1} - \frac{1}{\infty}\right] = R$$

or $$\lambda_s = \frac{1}{R} = \frac{1}{10967700} \text{ m}$$

$$= 911.6 \times 10^{-10} \text{m} = \mathbf{911.6\ Å}$$

Example 27. *Calculate the shortest wavelength in the Balmer series of hydrogen atom. In which region (infrared, visible, ultraviolet) of hydrogen spectrum does this wavelength lie ?* [CBSE OD 15]

Solution. For shortest wavelength of Balmer series, $n_1 = 2, n_2 = \infty$

$$\therefore \quad \frac{1}{\lambda_s} = R\left[\frac{1}{2^2} - \frac{1}{\infty}\right] = \frac{R}{4}$$

or $$\lambda_s = \frac{4}{R} = \frac{4}{1.1 \times 10^7} \text{m}$$

$$= 3.637 \times 10^{-7} \text{m} = \mathbf{3637\ Å}$$

This wavelength lies in the ultraviolet region of the em spectrum.

Example 28. *Using Rydberg formula, calculate the longest wavelengths belonging to Lyman and Balmer series. In which region of hydrogen spectrum do these transitions lie ? [Given $R = 1.1 \times 10^7 \text{m}^{-1}$]* [CBSE F 15]

Solution. Rydberg's formula is $\frac{1}{\lambda_{if}} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$

For longest wavelength of Lyman series : $n_i = 2, \ n_f = 1$

$$\therefore \quad \frac{1}{\lambda_{21}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}R$$

$$\therefore \quad \lambda_{21} = \frac{4}{3R} = \frac{4}{3 \times 1.1 \times 10^7} \text{m}$$

$$= 1.21 \times 10^{-7} \text{m} = \mathbf{121\ nm}$$

This transition lies in the ultraviolet region.

For longest wavelength of Balmer series : $n_i = 3, \ n_f = 2$

$$\therefore \quad \lambda_{32} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = R\left[\frac{1}{4} - \frac{1}{9}\right] = \frac{5}{36}R$$

or $$\lambda_{32} = \frac{36}{5R} = \frac{36}{5 \times 1.1 \times 10^7} \text{m}$$

$$= 6.545 \times 10^{-7} \text{m}$$

$$\simeq \mathbf{655\ nm}$$

This transition lies in the visible region.

Example 29. *The wavelength of H_α-line of the Balmer series is 6553 Å. Calculate the value of Rydberg constant.*

Solution. The Balmer series is given by the wave number

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

For H_α line, $n = 3$ and $\lambda = 6553$ Å, therefore,

$$\frac{1}{6553 \times 10^{-10}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36} R$$

$$\therefore \quad R = \frac{36 \times 10^{10}}{5 \times 6553} = \mathbf{10980000\ m^{-1}}.$$

Example 30. *A 12.9 eV beam of electrons is used to bombard gaseous hydrogen at room temperature. Upto which energy level the hydrogen atoms would be excited ? Calculate the wavelength of the first member of Paschen series and first member of Balmer series.* [CBSE D 14]

Solution. $E_n = -\dfrac{13.6}{n^2}$ eV

Energy required to excite the hydrogen atoms from ground state ($n = 1$) to the third excited state ($n = 4$) is

$$\Delta E = E_4 - E_1 = -\frac{13.6}{4^2} - \left(-\frac{13.6}{1^2}\right)$$

$$= +12.75 \text{ eV}$$

Hence, hydrogen atoms would be excited upto 4th energy level ($n = 4$) or third excited state as shown in the figure.

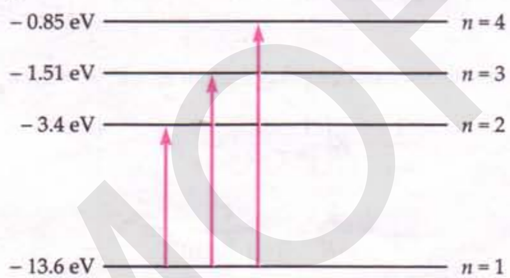

Fig. 12.15

For first line of Paschen series : $n_i = 4$ and $n_f = 3$, so

$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

or $$\frac{1}{\lambda_{43}} = 1.097 \times 10^7 \left[\frac{1}{3^2} - \frac{1}{4^2}\right]$$

$$= \frac{1}{0.912 \times 10^{-7}} \times \frac{7}{144}$$

or $$\lambda_{43} = \frac{0.912 \times 144 \times 10^{-7}}{7} \text{ m} = \mathbf{1875\ nm}.$$

For first line of Balmer series : $n_i = 3$ and $n_f = 2$, so

$$\frac{1}{\lambda_{32}} = 1.097 \times 10^7 \left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= \frac{1}{0.912 \times 10^{-7}} \times \frac{5}{36}$$

or $$\lambda_{32} = \frac{0.912 \times 36 \times 10^{-7}}{5} \text{ m} = \mathbf{656.3\ nm}.$$

Example 31. *Which state of the triply ionised beryllium (Be^{3+}) has the same orbital radius as that of the ground state of hydrogen ?* [NCERT]

Solution. Radius of nth orbit is given by

$$r_n = \frac{n^2 h^2}{4\pi^2 mkZe^2} \quad i.e., \quad r_n \propto \frac{n^2}{Z}$$

Let $r_n\ (Be^{3+}) = r_1(H)$

$$\therefore \quad \left[\frac{n^2}{Z}\right]_{Be^{3+}} = \left[\frac{n^2}{Z}\right]_H$$

or $$\frac{n^2}{4} = \frac{1^2}{1} \quad \text{or} \quad n = \mathbf{2}.$$

Example 32. *Which level of the double ionised lithium (Li^{2+}) has the same energy as the ground state energy of the hydrogen atom ? Compare the orbital radius of the two levels.* [NCERT]

Solution. Energy of the electron in nth orbit is given by

$$E = \frac{2\pi^2 mk^2 e^4}{h^2} \cdot \frac{Z^2}{n^2}$$

i.e., $$E_n \propto \frac{Z^2}{n^2}$$

Let $E_n\ (Li^{2+}) = E_1\ (H)$

$$\therefore \quad \left[\frac{Z^2}{n^2}\right]_{Li^{2+}} = \left[\frac{Z^2}{n^2}\right]_H$$

or $$\frac{3^2}{n^2} = \frac{1^2}{1^2} \quad \text{or} \quad n = \mathbf{3}.$$

Example 33. *A free electron of energy 1.4 eV collides with a H^+ ion. As a result of collision, a hydrogen atom in the ground state is formed and a photon is released. What is the wavelength of the emitted radiation ? In which part of the electromagentic spectrum does this spectrum lie ? Given ionisation potential of hydrogen = 13.6 eV.*

Solution. I.E. energy of hydrogen, $E_1 = -13.6$ eV

Energy of free electron, $E_2 = 1.4$ eV

Energy of emitted photon is

$$h\nu = E_2 - E_1 = [1.4 - (-13.6)] = 15.0 \text{ eV}$$

or $$\frac{hc}{\lambda} = 15 \times 1.6 \times 10^{-19} \text{ J}$$

∴ Wavelength of emitted photon is

$$\lambda = \frac{hc}{15 \times 1.6 \times 10^{-19}} \text{ m}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{15 \times 1.6 \times 10^{-19}} \text{ m}$$

$$= 825 \times 10^{-10} \text{m} = \mathbf{825 \text{ Å}}$$

This wavelength lies in the ultraviolet region.

Problems For Practice

1. At what speed must the electron revolve around the nucleus of the hydrogen atom so that it may not be pulled into the nucleus by electrostatic attraction ? Given $r = 0.5 \times 10^{-10}$ m, $m_e = 9.1 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ C. (Ans. $2.25 \times 10^6 \text{ms}^{-1}$)
2. Calculate the energy of the hydrogen atom in the states $n = 4$ and $n = 2$. Determine the frequency and wavelength of the emitted radiation in a transition from $n = 4$ to $n = 2$ state. Is this radiation visible ? (Ans. 0.85 eV, 3.4 eV, 6.16×10^{14} Hz, 4.87×10^{-7}m ; Yes)
3. Calculate (*i*) the radius of the second orbit of hydrogen atom, and (*ii*) total energy of electron moving in the second orbit. (Ans. 2.12 Å, – 3.4 eV)
4. Calculate the frequency of the photon, which can excite the electron to – 3.4 eV from – 13.6 eV. **[CBSE OD 2001]** (Ans. 2.47×10^{15} Hz)
5. The ground state energy of hydrogen atom is – 13.6 eV. If an electron makes a transition from an energy level – 0.85 eV to – 3.4 eV, calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum, does this wavelength belong ? **[CBSE D 01 ; OD12]** (Ans. 4853 Å, Balmer series)
6. Show that the ionisation potential of hydrogen atom is 13.6 volt.
7. The ground state energy of hydrogen atom is – 13.6 eV. If an electron makes a transition from an energy level – 0.85 eV to – 1.51 eV, calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong ? **[CBSE OD 12]** (Ans. 18840 Å, Paschen series)
8. The ground state energy of hydrogen atom is – 13.6 eV.
   (*i*) What are the potential energy and kinetic energy of an electron in the 3rd excited state ?
   (*ii*) If the electron jumps to the ground state from the third excited state, calculate the frequency of photon emitted. **[CBSE Sample Paper 11]**
   (Ans. (*i*) – 1.7 eV, 0.85 eV (*ii*) 3×10^{15} Hz)
9. The wavelength of H_α line in Balmer series is 6563 Å. Compute the wavelength of H_β line of Balmer series. (Ans. 4861.5 Å)
10. If the wavelength of the first member of Balmer series in hydrogen spectrum is 6563 Å, calculate the wavelength of the first member of Lyman series in the same spectrum. (Ans. 1215.14 Å)
11. The second member of Lyman's series in hydrogen spectrum has a wavelength of 5400 Å. Calculate the wavelength of the first member. (Ans. 6400 Å)
12. A photon of energy 12.09 eV is absorbed by an electron in ground state of a hydrogen atoms. What will be the energy level of electron ? The energy of electron in the ground state of hydrogen atom is – 13.6 eV (Ans. $n = 3$)
13. The period of revolution of the electron in the third orbit in a hydrogen atom is 4.132×10^{-15}s. Hence find the period in the 5th Bohr orbit. (Ans. 19.13×10^{-15}s)
14. In a hydrogen atom, a transition takes place from $n = 3$ to $n = 2$ orbit. Calculate the wavelength of the emitted photon. Will the photon be visible ? To which spectral series will this photon belong ? Given $R = 1.097 \times 10^7 \text{m}^{-1}$. (Ans. 6563 Å, Yes, Balmer)
15. Show that the shortest wavelength lines in Lyman, Balmer and Paschen series have their wavelengths in the ratio 1 : 4 : 9.
16. The short wavelength limit for the Lyman series of the hydrogen spectrum is 913.4 Å. Calculate the short wavelength limit for Balmer series of hydrogen spectrum. **[CBSE SP 15]** (Ans. 3653.6 Å)
17. A 12.3 eV electron beam is used to bombard gaseous hydrogen at room temperature. Upto which energy level the hydrogen atoms would be excited ? Calculate the wavelengths of the second member of Lyman series and second member of Balmer series. **[CBSE D 14]**
   (Ans. $n = 3$, $\lambda_{31} = 102.5$ nm, $\lambda_{42} = 486$ nm)

HINTS

1. Here $\frac{mv^2}{r} = k\frac{q_1 q_2}{r^2}$

or $v^2 = k\frac{q_1 q_2}{mr} = \frac{9\times10^9 \times 1.6\times10^{-19}\times1.6\times10^{-19}}{9.1\times10^{-31}\times0.5\times10^{-10}}$

$= \frac{4.8\times10^6}{\sqrt{4.55}}\ \text{ms}^{-1}$

$v = \mathbf{2.25\times10^6\ ms^{-1}}$.

3. (*i*) $r_n = \frac{n^2h^2}{4\pi^2 mke^2}$

$\therefore r_2 = \frac{2^2\times(6.63\times10^{-34})^2}{4\times9.87\times9.1\times10^{-31}\times9\times10^9\times(1.6\times10^{-19})^2}$

$= 2.12\times10^{-10}\text{m} = \mathbf{2.12\ Å}$.

(*ii*) $E_n = -\frac{2\pi^2 mk^2e^4}{n^2h^2}$

$E_2 = -\frac{2\times9.87\times9.1\times10^{-31}\times(9\times10^9)^2\times(1.6\times10^{-19})^4}{2^2\times(6.63\times10^{-34})^2}$

$= -5.44\times10^{-19}\text{J} = \mathbf{-3.4\ eV}$.

4. Energy of photon, $h\nu = E_2 - E_1$

Frequency, $\nu = \frac{E_2 - E_1}{h}$

$= \frac{[-3.4-(-13.6)]\times1.6\times10^{-19}}{6.6\times10^{-34}}$

$= \frac{10.2\times1.6\times10^{15}}{6.6} = \mathbf{2.47\times10^{15}\ Hz}$

5. Energy of emitted photon

$= E_2 - E_1 = -0.85-(-3.4) = 2.55\,\text{eV}$

$= 2.55\times1.6\times10^{-19}\text{J}$

As

$\lambda = \frac{hc}{E_2-E_1} = \frac{6.6\times10^{-34}\times3\times10^8}{2.55\times1.6\times10^{-19}}$

$= 4.853\times10^{-7}\text{m} = \mathbf{4853\ Å}$

This wavelength belongs to Balmer series of hydrogen spectrum.

6. I.E. of hydrogen

$= E_\infty - E_1 = \frac{2\pi^2mk^2e^4}{h^2}\left[\frac{1}{1^2}-\frac{1}{\infty}\right]$

$= \frac{2\times9.87\times9.1\times10^{-31}\times(9\times10^9)^2\times(1.6\times10^{-19})^4}{(6.63\times10^{-34})^2}\text{J}$

$= 21.7\times10^{-19}\text{ J} = \frac{21.7\times10^{-19}}{1.6\times10^{-19}}\text{ eV} = 13.6\,\text{eV}$

$\therefore$ Ionisation potential of hydrogen = **13.6 V.**

7. Here $\Delta E = E_2 - E_1 = -0.85-(-1.51) = 0.66\text{ eV}$

$= 0.66\times1.6\times10^{-19}\text{J}$

$\therefore \lambda = \frac{hc}{\Delta E} = \frac{6.63\times10^{-34}\times3\times10^8}{0.66\times1.6\times10^{-19}}\text{m}$

$= 18.84\times10^{-7}\text{m} = \mathbf{18840\ Å}$

This wavelength belongs to the Paschen series of the hydrogen spectrum.

8. Given $E_1 = -13.6\,\text{eV}$

For third excited state, $n = 4$

$\therefore E_4 = \frac{E_1}{4^2} = -\frac{13.6}{16}\text{eV} = -0.85\,\text{eV}$

$\therefore$ K.E. $= -E_4 = \mathbf{0.85\ eV}$

and P.E. $= 2E_4 = \mathbf{-1.7\ eV}$

$\Delta E = E_4 - E_1$

$= [-0.85-(-13.6)]\,\text{eV} = 12.75\,\text{eV}$

$\therefore \nu = \frac{\Delta E}{h} = \frac{12.75\times1.6\times10^{-19}}{6.63\times10^{-34}}$

$= \mathbf{3\times10^{15}\ Hz}$

9. For H_α line of Balmer series,

$\frac{1}{\lambda_1} = R\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = \frac{5}{36}R$

For H_β line of Balmer series,

$\frac{1}{\lambda_2} = R\left[\frac{1}{2^2}-\frac{1}{4^2}\right] = \frac{3}{16}R$

$\therefore \frac{\lambda_2}{\lambda_1} = \frac{5}{36}\times\frac{16}{3} = \frac{20}{27}$

or $\lambda_2 = \frac{20}{27}\times\lambda_1$

$= \frac{20}{27}\times6563 = \mathbf{4861.5\ Å}$

12. $E_n - E_1 = h\nu$

or $E_n = h\nu + E_1 = 12.09 - 13.6 = -1.51\,\text{eV}$

As $E_n = \frac{E_1}{n^2}$

$\therefore n^2 = \frac{E_1}{E_n} = \frac{-13.6}{-1.51} \simeq 9$ or $n = 3$.

13. $T_n = T_1 n^3$ *i.e.,* $T_n \propto n^3$

$\therefore \frac{T_5}{T_3} = \frac{5^3}{3^3} = \frac{125}{27}$

$T_5 = \frac{125}{27}\times T_3$

$= \frac{125}{27}\times4.132\times10^{-15}$

$= \mathbf{19.13\times10^{-15}\ s.}$

14. $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = 1.097 \times 10^7 \left[\frac{1}{2^2} - \frac{1}{3^2}\right]$

$= 1.097 \times 10^7 \times \frac{5}{36}$

or $\lambda = \frac{36}{1.097 \times 10^7 \times 5}$ m $= 6.563 \times 10^{-7}$ m $=$ **6563 Å.**

This wavelength lies in the visible (red) part of the spectrum, hence the photon is visible. It is the first member of the Balmer series.

15. For shortest wavelength of Lyman series,

$n_1 = 1, \quad n_2 = \infty$

$\therefore \quad \frac{1}{\lambda_{LS}} = R\left[\frac{1}{1^2} - \frac{1}{\infty}\right] = R$ or $\lambda_{LS} = \frac{1}{R}$

For shortest wavelength of Balmer series,

$n_1 = 2, \quad n_2 = \infty$

$\therefore \quad \frac{1}{\lambda_{BS}} = R\left[\frac{1}{2^2} - \frac{1}{\infty}\right] = \frac{R}{4}$ or $\lambda_{BS} = \frac{4}{R}$

For shortest wavelength of Paschen series,

$n_1 = 3, \quad n_2 = \infty$

$\frac{1}{\lambda_{PS}} = R\left[\frac{1}{9} - \frac{1}{\infty}\right] = \frac{R}{9}$ or $\lambda_{PS} = \frac{9}{R}$

Hence $\lambda_{LS} : \lambda_{BS} : \lambda_{PS} = \mathbf{1 : 4 : 9}$.

16. From the above problem, we have

$\lambda_{LS} = \frac{1}{R}$ and $\lambda_{BS} = \frac{4}{R}$

Clearly, $\lambda_{BS} = 4\lambda_{LS} = 4 \times 913.4 =$ **3653.6 Å.**

17. $E_n = -\frac{13.6}{n^2}$eV. Energy required to excite the hydrogen atoms from ground state ($n = 1$) to the second excited state ($n = 3$) is

$\Delta E = E_3 - E_1$

$= -\frac{13.6}{3^2} - \left(-\frac{13.6}{1^2}\right) = -1.51 - (-13.6) = 12.09$ eV

Hence, hydrogen atoms would be excited upto third energy level ($n = 3$) or second excited state with an electron beam of 12.3 eV.

For second member of Lyman series : $n_i = 3$ and $n_f = 1$, so

$\frac{1}{\lambda_{31}} = 1.097 \times 10^7 \left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{1}{0.912 \times 10^{-7}} \times \frac{8}{9}$

or $\lambda_{31} = \frac{0.912 \times 9 \times 10^{-7}}{8}$ m

$= 1.025 \times 10^{-7}$ m $=$ **102.5 nm.**

For second member of Balmer series : $n_i = 4$ and $n_f = 2$, so

$\frac{1}{\lambda_{42}} = 1.097 \times 10^7 \left[\frac{1}{2^2} - \frac{1}{4^2}\right]$

$= \frac{1}{0.912 \times 10^{-7}} \times \frac{3}{16}$

or $\lambda_{42} = \frac{0.912 \times 16 \times 10^{-7}}{3}$ m

$= 4.86 \times 10^{-7}$ m $=$ **486 nm.**

Very Short Answer Conceptual Problems

Problem 1. **Why Thomson's model of the atom is known as a plum pudding model ?**

Solution. This is because in Thomson's model the electrons are assumed to be uniformly embedded in a sphere of positively charged matter like the plums are arranged in a pudding.

Problem 2. **Why did Thomson's atomic model fail ?** [Punjab 04]

Solution. Thomson model failed to explain the scattering of α-particles through large angles in Rutherford's experiment.

Problem 3. **Write two important inferences drawn from Rutherford's alpha particle scattering experiment.** [CBSE OD 05C]

Solution. (*i*) The most of the mass and the entire positive charge of an atom is concentrated in a very small volume of the atom, called nucleus.

(*ii*) The nuclear radius is about 1/10,000 of the atomic radius.

Problem 4. **Why is it that mass of the nucleus does not enter the formula for impact parameter, but its charge does ?**

Solution. The scattering occurs due to the electrostatic field of the nucleus. That is why charge of nucleus enters the expression for the impact parameter.

Problem 5. **Why do we use a very thin gold foil in Rutherford's α-particle scattering experiment ?**

Solution. In thick foil, the entire kinetic energy of the α-particles will be absorbed and so α-particles will not be able to penetrate through the foil.

Problem 6. **The kinetic energy of α-particle incident on gold foil is doubled. How does the distance of closest approach change ?** [CBSE OD 12]

Solution. As the distance of closest approach is inversely proportional to the kinetic energy of the incident α -particle, so the distance of closest approach is halved when the kinetic energy of α-particle is doubled.

Problem 7. **The large angle scattering is possible only due to nucleus. Why ?**

Solution. α-particles can be scattered through large angles only if they collide against a positively charged heavy particle such as a nucleus.

Problem 8. **The Rutherford α-particle scattering experiment shows that most of the α-particles pass through almost unscattered while some of them are scattered through large angles. What information does it give about the structure of atom ?**

Solution. (*i*) As most of α-particles pass straight, it indicates that most of the space in the atom is empty.

(*ii*) The large angle scattering indicates that most of the mass and the entire positive charge of the atom is concentrated in a small central core, called nucleus.

Problem 9. **Why is electron revolving round the nucleus of an atom ?** [Haryana 11]

Solution. If the electrons were stationary, they would fall into the nucleus due to the electrostatic attraction and the atom would be unstable.

Problem 10. **In the Rutherford scattering experiment the distance of closest approach for an α-particle is d_0. If α-particle is replaced by a proton, how much kinetic energy in comparison to α-particle will it require to have the same distance of closest approach d_0 ?** [CBSE F 09]

Solution. At the distance of closest approach,

$$K_\alpha = \frac{kZe.2e}{d_0}$$

and
$$K_p = \frac{kZe.e}{d_0}$$

$$\therefore \quad K_p = \frac{1}{2} K_\alpha$$

Thus, a proton would need half the initial K.E. of that of an α-particle for the distance d_0.

Problem 11. **What is the significance of the negative energy of electron in the orbit ?** [Haryana 11]

Solution. This signifies that the electron is bound to the nucleus. Due to electrostatic attraction between electron and nucleus, the P.E. is negative and is greater than K.E. of electron. Total energy of electron is negative. It cannot escape from the atom.

Problem 12. **What are stationary orbits ?**

Solution. Bohr postulated that electrons can revolve around the nucleus in certain discrete, non-radiating orbits in which the angular momentum of an electron is an integral multiple of $h/2\pi$. Such orbits are called stationary orbits.

Problem 13. **State Bohr's quantisation condition in terms of de-Broglie wavelength.**

Solution. According to Bohr's quantisation condition, only such circular orbits are allowed as stationary states of an electron which contain an integral multiple of de-Broglie wavelengths.

Problem 14. **State Bohr's postulate of quantization of angular momentum of the orbiting electron in hydrogen atom.** [CBSE D 09C]

Solution. According to Bohr's theory, only such orbits are permissible for the motion of an electron around the nucleus in which the angular momentum of an electron is integral multiple of $h/2\pi$.

$$L = mvr = \frac{nh}{2\pi}, \qquad n = 1, 2, 3, \ldots$$

Here n is the principal quantum number.

Problem 15. **When an electron falls from a higher energy to a lower energy level, the difference in the energies appears in the form of electromagnetic radiation. Why cannot it be emitted as other forms of energy ?** [Exemplar Problem]

Solution. This is because electrons interact only electromagnetically.

Problem 16. **Consider two different hydrogen atoms. The electron in each atom is in an excited state. Is it possible for the electrons to have different energies, but the same orbital angular momentum according to the Bohr model ?** [Exemplar Problem]

Solution. No, because according to Bohr model, $E_n = -\frac{13.6}{n^2}$ and electrons having different energies belong to different levels having different values of n. So, their angular momenta will be different, as

$$L = mvr = \frac{nh}{2\pi}.$$

Problem 17. **With increasing quantum number *n*, state whether the energy difference between adjacent levels increases or decreases ?**

Solution. The energy difference between two adjacent levels decreases with the increase in the value of n.

Problem 18. **How much is the energy possessed by an electron for $n = \infty$?**

Solution. Zero.

For $n = \infty$, $E_n = -\frac{13.6}{n^2}\text{ eV} = 0.$

Problem 19. **In a hydrogen atom, if the electron is replaced by a particle which is 200 times heavier but has the same charge, how would its radius change ?** [CBSE F 08]

Solution. Radius, $r \propto \frac{1}{m}$

When the electron is replaced by a 200 times heavier particle, the radius decreases to $\frac{1}{200}$ of the original radius.

Problem 20. **The short wavelength limits of the Lyman, Paschen and Balmer series, in the hydrogen spectrum, are denoted by λ_L, λ_P and λ_B respectively. Arrange these wavelengths in increasing order.** [CBSE Sample Paper 11]

Solution. $\lambda_L < \lambda_B < \lambda_P$.

Problem 21. **Find the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its (*i*) second permitted energy level to the first level, and (*ii*) the highest permitted energy level to the first permitted level.** [CBSE OD 10]

Solution. Energy of a photon

$$= h\nu\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$$\therefore \quad \frac{(h\nu)_{2\to1}}{(h\nu)_{\infty\to1}}=\frac{\left(\frac{1}{1^2}-\frac{1}{2^2}\right)}{\left(\frac{1}{1^2}-\frac{1}{\infty^2}\right)}=\frac{3}{4}=\mathbf{3:4.}$$

Problem 22. **Calculate the ratio of energies of photons produced due to transition of electron of hydrogen atom from its, (*i*) second permitted energy level to the first level, and (*ii*) highest permitted energy level to the second permitted level.** [CBSE SP 08]

Solution. (*i*) $\quad \nu_1 \propto \left[\frac{1}{1^2}-\frac{1}{2^2}\right]$ or $\nu_1 \propto \frac{3}{4}$

(*ii*) $\quad \nu_2 \propto \left[\frac{1}{2^2}-\frac{1}{\infty^2}\right]$ or $\nu_2 \propto \frac{1}{4}$

$$\therefore \quad \frac{E_1}{E_2}=\frac{\nu_1}{\nu_2}=\frac{3}{4}\times\frac{4}{1}= \mathbf{3:1.}$$

Short Answer Conceptual Problems

Problem 1. **In the Rutherford's nuclear model of the atom, the nucleus (radius about 10^{-15} m) is analogous to the sun about which the electrons move in orbit (radius $\approx 10^{-10}$ m) like the earth orbits around the sun. If the dimensions of the solar system had the same proportions as those of the atom, would the earth be closer to or farther away from the sun than actually it is ? The radius of earth orbit is about 1.5×10^{11} m. The radius of sun is taken as 7×10^{8} m** [NCERT]

Solution. The ratio of the radius of electron's orbit to the radius of nucleus is 10^{-10} m / 10^{-15} m) = 10^5. That is the radius of the electron's orbit is 10^5 times larger than the radius of nucleus. If the radius of the earth's orbit around the sun were 10^5 times larger than the radius of the sun, the radius of the earth's orbit would be $10^5\times7\times10^8$ m $=7\times10^{13}$ m. This is more than 100 times greater than the actual orbital radius. Thus the earth would be much farther away from the sun.

It implies that an atom contains a much greater fraction of empty space than our solar system does.

Problem 2. **The trajectories, traced by different α-particles, in Geiger-Marsden experiment were observed as shown in Fig. 12.16.**

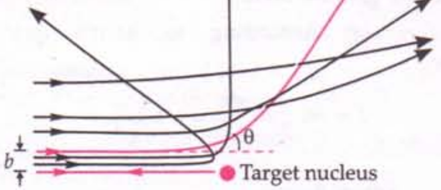

Fig. 12.16

What names are given to the symbols '*b*' and 'θ' shown here ?

What can we say about the values of '*b*' for (*i*) $\theta \simeq 0°$ and (*ii*) $\theta \simeq \pi$ radians ? [CBSE D 08C]

Solution. The symbol '*b*' represents *impact parameter* and 'θ' represents the *scattering angle*.

(*i*) When $\theta = 0°$, the impact parameter b is large and the α-particle passes almost undeflected.

(*ii*) When $\theta \simeq \pi$ radians, the impact parameter $b = 0$ and the α-particle is reversed back along its original path.

Problem 3. **In the study of Geiger-Marsden experiment on scattering of α particles by a thin foil of gold, draw the trajectory of α-particles in the Coulomb field of target nucleus. Explain briefly how one gets the information on the size of the nucleus from this study.** [CBSE D 15]

Solution. See Fig. 12.16 Only a very small fraction of α-particles gets rebounded. This suggests that all the positive charge and the mass of the atom is concentrated in a very small region called the nucleus of the atom and it gives an idea of the size of the nucleus.

Problem 4. **Explain why the spectrum of hydrogen atom has many lines, although a hydrogen atom contains only one electron.**

Solution. A source of hydrogen spectrum has billions of hydrogen atoms. Each hydrogen atom has many stationary states. All possible transitions can occur from any higher level to any lower level. This gives rise to a large number of spectral lines.

Problem 5. Using Rutherford model of the atom, derive the expression for the total energy of the electron in hydrogen atom. What is the significance of total negative energy possessed by the electron ? [CBSE OD 14]

Solution. According to Rutherford's model for H-atom,

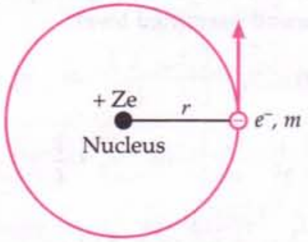

Fig. 12.17

Centripetal force on electron = Electrostatic attraction between electron and nucleus

or $$\frac{mv^2}{r} = k\frac{Ze.e}{r^2}$$

or $$mv^2 = \frac{kZe^2}{r}$$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{kZe^2}{2r}$$

$$\text{P.E.} = k\frac{q_1q_2}{r} = \frac{kZe(-e)}{r} = -\frac{kZe^2}{r}$$

Total energy = K.E. + P.E.

or $$E_n = \frac{kZe^2}{2r} - \frac{kZe^2}{r}$$

$$= -\frac{kZe^2}{2r} = -\frac{1}{8\pi\varepsilon_0}\frac{Ze^2}{r}$$

The negative sign indicates that the electron is bound to the nucleus by means of electrostatic attraction.

Problem 6. Using Bohr's postulates of the atomic model, derive the expression for radius of n^{th} electron orbit. Hence obtain the expression for Bohr's radius. Show graphically the (nature of) variation of the radius of the orbit with the principal quantum number n. [CBSE SP 11 ; OD 14]

Solution. For a circular orbit of the electron,

$$\frac{mv^2}{r} = \frac{kZe\cdot e}{r^2} = \frac{kZe^2}{r^2}$$

or $$mv^2 = \frac{kZe^2}{r}$$

or $$r = \frac{kZe^2}{mv^2} \quad ...(i)$$

Using Bohr's quantisation condition for angular momentum,

$$L = mvr = \frac{nh}{2\pi}$$

or $$r = \frac{nh}{2\pi mv} \quad ...(ii)$$

From (*i*) and (*ii*),

$$\frac{kZe^2}{mv^2} = \frac{nh}{2\pi mv}$$

or $$v = \frac{2\pi kZe^2}{nh}$$

$$\therefore \quad r = \frac{nh}{2\pi m}\cdot\frac{nh}{2\pi kZe^2}$$

$$= \frac{n^2h^2}{4\pi^2 mkZe^2}.$$

Bohr's radius, $$r_0 = \frac{h^2}{4\pi^2 mke^2} \quad [n = 1, Z = 1]$$

As $r \propto n^2$, the graph of r versus n is a parabola as shown in Fig. 12.18.

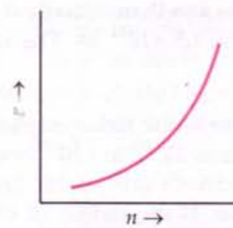

Fig. 12.18

Problem 7. (*a*) Using Bohr's second postulate of quantization of orbital angular momentum, show that the circumference of the electron in the *n*th orbital state in hydrogen atom is *n* times the de-Broglie wavelength associated with it.

(*b*) The electron in hydrogen atom is initially in the third excited state. What is the maximum number of spectral lines which can be emitted when it finally moves to the ground state ? [CBSE D 12 ; OD 13C]

Solution. (*a*) According to Bohr's quantisation condition,

$$L = mvr_n = \frac{nh}{2\pi}, \quad n = 1,2,3,...$$

or $$2\pi r_n = n\frac{h}{mv}$$

But $\frac{h}{mv}$ = de-Broglie wavelength (λ)

$$\therefore \quad 2\pi r_n = n\lambda$$

Thus the circumference of nth orbit contains exactly n de-Broglie wavelengths.

(*b*) For third excited state, $n = 4$

For ground state, $n = 1$

Hence, the possible transitions are

$$n_i = 4 \text{ to } n_f = 3, 2, 1$$

$$n_i = 3 \text{ to } n_f = 2, 1$$

$$n_i = 2 \text{ to } n_f = 1$$

$\therefore$ Total number of transitions = 6, as shown in Fig. 12.19.

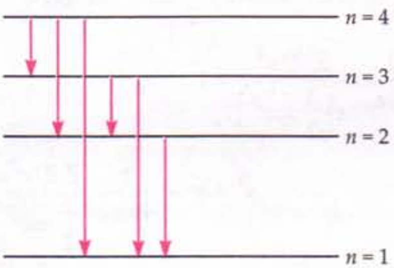

Fig. 12.19

Problem 8. **(*a*) The energy levels of an atom are as shown below. Which of them will result in the transition of a photon of wavelength 275 nm ?**

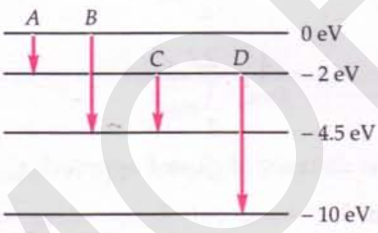

Fig. 12.20

(*b*) Which transition corresponds to emission of radiation of (*i*) maximum wavelength and (*ii*) minimum wavelength ? [CBSE D 09, 11 ; F 13]

Solution. (*a*) The energy E of a photon of wavelength 275 nm is given by

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{275 \times 10^{-9}} \text{ J}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{275 \times 10^{-9} \times 1.6 \times 10^{-19}} \text{ eV} = 4.5 \text{ eV}$$

This energy corresponds to the transition B for which the energy change $= 0 - (-4.5) = 4.5$ eV.

(*b*) Energy of emitted photon,

$$E = \frac{hc}{\lambda} \propto \frac{1}{\lambda}$$

$$\therefore \quad \lambda_{max} \propto \frac{1}{E_{min}}$$

and $$\lambda_{min} \propto \frac{1}{E_{max}}$$

(*i*) Transition A, for which the energy emission is minimum, corresponds to the emission of radiation of *maximum wavelength*.

(*ii*) Transition D, for which the energy emission is maximum, corresponds to the emission of radiation of *minimum wavelength*.

Problem 9. **Photons, with a continuous range of frequencies, are made to pass through a simple of rarefied hydrogen. The transitions, shown in Fig. 12.21, indicate three of the spectral absorption lines in the continuous spectrum.**

(*i*) Identify the spectral series, of the hydrogen emission spectrum, to which each of these three lines correspond.

(*ii*) Which of these lines corresponds to the absorption of radiation of maximum wavelength ?

[CBSE D 09C]

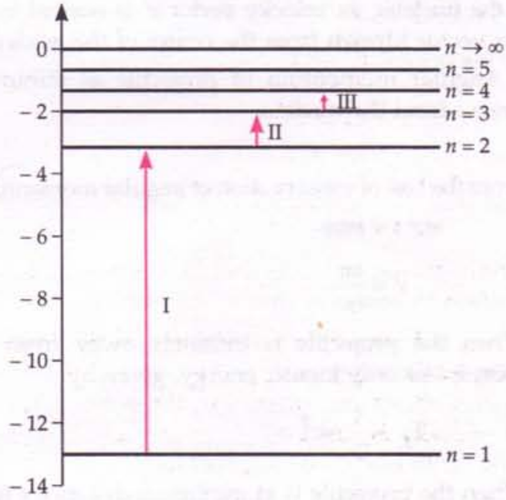

Fig. 12.21

Solution. (*i*) Line I → Lyman series.

Line II → Balmer series and Line III → Paschen series

(*ii*) Line III corresponds to absorption of photon of minimum energy and hence of maximum wavelength.

HOTS Problems on Higher Order Thinking Skills

Problem 1. *A projectile of mass m, charge Z′, initial speed v and impact parameter b is scattered by a heavy nucleus of charge Z. Use angular momentum and energy conservation to obtain a formula connecting the minimum distance s of the projectile from the nucleus to these parameters. Show that for b = 0, s reduces to the distance of closest approach r_0. [Ignore size of the nucleus and its recoil motion].*

Solution. Charge on the heavy nucleus = Ze

Charge on the projectile $= Z'e$

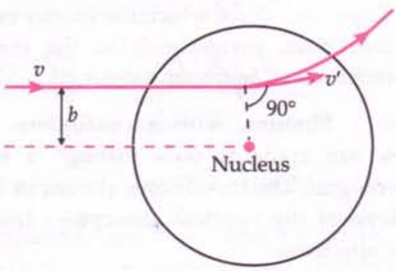

Fig. 12.22

Angular momentum of the projectile when it lies at infinity from the nucleus

$$= mvb$$

When the projectile lies at the minimum distance s from the nucleus, its velocity vector v' is normal to the radius vector (drawn from the centre of the nucleus).

∴ Angular momentum of projectile at minimum distance s from the nucleus

$$= mv's$$

From the law of conservation of angular momentum,

$$mv's = mvb$$

or $$v' = \frac{vb}{s} \quad ...(1)$$

When the projectile is infinitely away from the nucleus, it has only kinetic energy, given by

$$E_K = \frac{1}{2}mv^2$$

When the projectile is at minimum distance s from the nucleus, it has both kinetic and potential energies, given by

$$E'_K = \frac{1}{2}mv'^2$$

and $$E_P = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze.Z'e}{s}$$

$$\therefore \quad E'_K + E_P = \frac{1}{2}mv'^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{ZZ'e^2}{s}$$

From the law of the conservation of energy

$$E'_K + E'_P = E_K$$

or $$\frac{1}{2}mv'^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{ZZ'e^2}{s} = \frac{1}{2}mv^2$$

or $$\frac{1}{2}m.\frac{v^2b^2}{s^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{ZZ'e^2}{s} = \frac{1}{2}mv^2 \quad \text{[Using (1)]}$$

Dividing both sides by $\frac{1}{2}mv^2$, we get

$$\frac{b^2}{s^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{ZZ'e^2}{s\left(\frac{1}{2}mv^2\right)} = 1$$

or $$s^2 = b^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{ZZ'e^2s}{\frac{1}{2}mv^2}$$

This is the required formula connecting the minimum distance with various parameters.

For a head-on collision, $b = 0$

$$\therefore \quad s^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{ZZ'e^2s}{\frac{1}{2}mv^2}$$

or $$s = \frac{1}{4\pi\varepsilon_0}\cdot\frac{ZZ'e^2}{\frac{1}{2}mv^2}$$

which is the distance of closest approach r_0.

Problem 2. *For scattering by an 'inverse-square' field (such as that produced by a charged nucleus in Rutherford's model) the relation between impact parameter b and the scattering angle θ is given by*

$$b = \frac{Ze^2 \cot\theta/2}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$$

(a) What is the scattering angle for b = 0 ?

(b) For a given impact parameter b, does the angle of deflection increase or decrease with increase in energy ?

(c) What is the impact parameter at which the scattering angle is 90° for Z = 79 and initial energy equal to 10 MeV ?

(d) *Why is it that the mass of the nucleus does not enter the formula above but its charge does ?*

(e) *For a given energy of the projectile, does the scattering angle increase or decrease with decrease in impact parameter ?*

Solution. In Rutherford's model, the impact parameter is given by

$$b = \frac{Ze^2 \cot\theta/2}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$$

(*a*) Given $b = 0$

$$\therefore \quad \frac{Ze^2 \cot\theta/2}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)} = 0$$

or $$\cot\frac{\theta}{2} = 0$$

[$\because$ All other quantities are finite]

$$\therefore \quad \frac{\theta}{2} = 90° \quad \text{or} \quad \theta = 180°$$

which is the value expected physically for a head-on collision.

(*b*) For a given value of *b*,

$$\frac{Ze^2 \cot\theta/2}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)} = \text{constant}$$

$\therefore$ As the energy $\left(\frac{1}{2}mv^2\right)$ increases, the value of $\cot\frac{\theta}{2}$ increases and hence the value of scattering angle θ decreases, as expected.

(*c*) Given $\theta = 90°$, $Z = 79$, $e = 1.6 \times 10^{-19}$C,

$$E = \frac{1}{2}mv^2 = 10 \text{ MeV}$$

$$= 10 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-12} \text{ J}$$

$$\therefore \quad b = \frac{Ze^2 \cot\theta/2}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$$

$$= \frac{9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2 \cot 45°}{1.6 \times 10^{-12}} \text{ m}$$

$$= 9 \times 79 \times 1.6 \times 10^{-11} \text{ m}$$

$$= 1137.6 \times 10^{-11} \text{ m} \simeq \mathbf{1.1 \times 10^{-14} \text{ m}}.$$

(*d*) It is the charge on the nucleus which provides the electrostatic field and due to which scattering of α-particles occurs. If $Z = 0$, then from given formula we have, $\theta = 0°$. This means that scattering does not occur when nucleus carries no charge. Mass of nucleus does not appear in the expression for *b*, because recoil of the nucleus is being ignored, *i.e.*, the nucleus is assumed to be at rest during its interaction with the α-particle.

(*e*) For a given energy $\left(\frac{1}{2}mv^2\right)$ of the projectile, the decrease in impact parameter *b* implies a decrease in the value of $\cot\theta/2$ and hence an increase in the scattering angle θ.

Problem 3. *The wavelength of the second line of the Balmer series in the hydrogen spectrum is 4861 Å. Calculate the wavelength of the first line.*

[CBSE Sample Paper 98]

Solution. The wavelengths λ_1 and λ_2 of the first and second lines of the Balmer series are given by

$$\frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}R$$

and $$\frac{1}{\lambda_2} = R\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3}{16}R$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{3}{16} \times \frac{36}{5} = \frac{27}{20}$$

or $$\lambda_1 = \frac{27}{20} \times \lambda_2 = \frac{27}{20} \times 4861 = \mathbf{6562 \text{ Å}}.$$

Problem 4. *The spectrum of a star in the visible and the ultraviolet region was observed and the wavelength of some of the lines that could be identified were found to be :*

824 Å, 970 Å, 1120 Å, 2504 Å, 5173 Å, 6100 Å

Which of these lines cannot belong to hydrogen atom spectrum ? (Given Rydberg's constant $R = 1.03 \times 10^7 m^{-1}$ and $\frac{1}{R} = 970$ Å). Support your answer with suitable calculations.

[CBSE Sample Paper 08]

Solution. The wavelengths of the spectral lines of hydrogen atom are given by the Rydberg formula,

$$\frac{1}{\lambda} = \bar{\nu} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

or $$\lambda = \frac{1/R}{\frac{1}{n_f^2} - \frac{1}{n_i^2}} = \frac{970 \text{ Å}}{\left[\frac{1}{n_f^2} - \frac{1}{n_f^2}\right]} \qquad \left[\because \frac{1}{R} = 970 \text{ Å}\right]$$

For Lyman series of hydrogen spectrum in the ultraviolet region, $n_f = 1$. Putting $n_i = 2, 3, ..., \infty$; we get different lines as follows :

$$\lambda = \frac{970 \text{ Å}}{(3/4)}, \frac{970 \text{ Å}}{(8/9)}, \frac{970 \text{ Å}}{(15/16)}, ... \frac{970 \text{ Å}}{1}$$

$$= 1293.3 \text{ Å}, 1091 \text{ Å}, 1034.6 \text{ Å}, ..., 970 \text{ Å}$$

For Balmer series of hydrogen spectrum in the visible region, $n_f = 2$. Putting $n_i = 3, 4, 5, ..., \infty$; we get different lines as follows :

$$\lambda = \frac{970 \text{ Å}}{\left[\frac{1}{3^2} - \frac{1}{2^2}\right]}, \frac{970 \text{ Å}}{\left[\frac{1}{4^2} - \frac{1}{2^2}\right]}, \frac{970 \text{ Å}}{\left[\frac{1}{5^2} - \frac{1}{2^2}\right]}, ..., \frac{970 \text{ Å}}{\left[\frac{1}{\infty^2} - \frac{1}{2^2}\right]}$$

$$= \frac{970 \text{ Å}}{\frac{5}{36}}, \frac{970 \text{ Å}}{\frac{3}{16}}, \frac{970 \text{ Å}}{\frac{21}{100}}, \frac{970 \text{ Å}}{\frac{1}{4}}$$

$$= 6984 \text{ Å}, 5173.3 \text{ Å}, 4619 \text{ Å}, ..., 3880 \text{ Å}$$

On comparing the wavelengths given in the question with the above determined wavelengths, we find that the following wavelengths cannot belong to the hydrogen atom spectrum :

$$834 \text{ Å}, 1120 \text{ Å}, 2504 \text{ Å}, 6100 \text{ Å}.$$

Problem 5. *The energy of an electron in an excited hydrogen atom is -3.4 eV. Calculate the angular momentum of the electron according to Bohr's theory.*

Given :

Rydberg's constant,

$$R = 1.09737 \times 10^7 \, m^{-1}$$

Planck's constant,

$$h = 6.63 \times 10^{-34} \, Js$$

Speed of light,

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

Solution. The energy of an electron in the nth orbit of hydrogen atoms is

$$E_n = -\frac{Rhc}{n^2}$$

$$= -\frac{1.09737 \times 10^7 \times 6.63 \times 10^{-34} \times 3 \times 10^8}{n^2} \text{ J}$$

$$= -\frac{1.09737 \times 6.63 \times 3 \times 10^{-19}}{n^2 \times 1.6 \times 10^{-19}} \text{eV}$$

$$= -\frac{13.6}{n^2} \text{ eV}$$

Given $E_n = -3.4$ eV

$$\therefore \quad -3.4 = -\frac{13.6}{n^2}$$

or $$n^2 = \frac{13.6}{3.4} = 4$$

or $$n = 2$$

From Bohr's quantum condition of angular momentum,

Angular momentum,

$$mvr = \frac{nh}{2\pi}$$

$$= \frac{2 \times 6.63 \times 10^{-34}}{2 \times 3.14} \text{ Js}$$

$$= \mathbf{2.1 \times 10^{-34} \text{ Js.}}$$

Problem 6. *The ground state energy of an atom is -13.6 eV. The photon emitted during the transition of electron from $n = 3$ to $n = 1$ state, is incident on a photosensitive material of unknown work function. The photoelectrons are emitted from the materials with a maximum kinetic energy of 9 eV. Calculate the threshold wavelength of the material used.*

[CBSE F 08]

Solution. For a transition from $n = 3$ to $n = 1$ state, the energy of the emitted photon,

$$h\nu = E_2 - E_1$$

$$= 13.6 \left[\frac{1}{1^2} - \frac{1}{3^2}\right] \text{eV} = 12.1 \text{ eV}.$$

From Einstein's photoelectric equation,

$$h\nu = K_{max} + W_0$$

$$\therefore \quad W_0 = h\nu - K_{max}$$

$$= 12.1 - 9 = 3.1 \text{ eV}$$

Threshold wavelength,

$$\lambda_0 = \frac{hc}{W_0}$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3.1 \times 1.6 \times 10^{-19}}$$

$$= \mathbf{4 \times 10^{-7} \text{ m.}}$$

Problem 7. *What is the minimum energy that must be given to a H-atom in ground state so that it can emit an H_γ in Balmer series ? If the angular momentum of the system is conserved, what would be the angular momentum of such H_γ photon ?* **[Exemplar Problem]**

Solution. H_γ in Balmer series corresponds to transition, $n=5$ to $n=2$. So the electron in ground state $n=1$ must first be put in state $n=5$.

Energy required $= E_1 - E_5 = 13.6 - 0.54 = 13.06$ eV

If angular momentum is conserved, angular momentum of photon

$$= \text{Change in angular momentum of electron}$$

$$= L_5 - L_2 = \frac{5h}{2\pi} - \frac{2h}{2\pi} = \frac{3h}{2\pi}$$

$$= 3 \times 1.06 \times 10^{-34} = \mathbf{3.18 \times 10^{-34}\ kg\ m^2 s^{-1}}.$$

Problem 8. *The electron, in a hydrogen atom, initially in a state of quantum number n_1 makes a transition to a state whose excitation energy, with respect to the ground state, is 10.2 eV. If the wavelength, associated with the photon emitted in this transition, is 487.5 nm, find the (i) energy in eV and (ii) value of the quantum number, n_1 of the electron in its initial state.* **[CBSE Sample Paper 13]**

Solution. Energy of an electron in the nth orbit of H-atom,

$$E_n = -\frac{13.6}{n^2}\ \text{eV}$$

$\therefore\ E_1 = -13.6\,\text{eV},\ E_2 = -3.4\,\text{eV},\ E_3 = -1.51\,\text{eV}, \ldots$

Clearly, $E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV.

Thus the energy state $n=2$ has an excitation energy of 10.2 eV with respect to the ground state. Hence the electron is making a transition from state $n = n_1$ to the state $n=2$, where $n_1 > 2$.

Now

$$E_{n_1} - E_2 = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{487.5 \times 10^{-9}}\ \text{J}$$

$$= \frac{6.63 \times 3 \times 10^{-17}}{487.5 \times 1.6 \times 10^{-19}}\ \text{eV} = 2.55\ \text{eV}$$

$$\therefore \quad E_{n_1} = E_2 + 2.55\ \text{eV}$$

$$= -3.4 + 2.55 = \mathbf{-0.85\ eV}$$

Also, $\quad E_{n_1} = -\frac{13.6}{n_1^2}\ \text{eV}$

$$\therefore \quad -\frac{13.6}{n_1^2} = -0.85$$

or $\quad n_1 = \sqrt{\frac{13.6}{0.85}} = \sqrt{16} = \mathbf{4}.$

GUIDELINES TO NCERT EXERCISES

12.1. *Choose the correct alternative from the clues given at the end of each statement :*

(*a*) *The size of the atom in Thomson's model is .... the atomic size in Rutherford's model.*

(much greater than, no different from, much less than).

(*b*) *In the ground state of ...... electrons are in stable equilibrium, while in ...... electrons always experience a net force.*

(Thomson's model, Rutherford's model).

(*c*) *A classical atom based on ...... is doomed to collapse.*

(Thomson's model, Rutherford's model)

(*d*) *An atom has a nearly continuous mass distribution in ...... but has a highly non-uniform mass distribution in ......*

(Thomson's model, Rutherford's model).

(*e*) *The positively charged part of the atom possesses most of the mass of the atom in ......*

(Rutherford's model, both the models).

Ans. (*a*) no different from

(*b*) Thomson's model, Rutherford's model

(*c*) Rutherford's model

(*d*) Thomson's model, Rutherford's model

(*e*) both the models

12.2. *Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K). What results do you expect ?*

Ans. Hydrogen nuclei (or protons) are much lighter than α-particles. So α-particles are not scattered by solid hydrogen. They pass through solid hydrogen almost undeflected from their paths.

12.3. *What is the shortest wavelength present in the Paschen series of spectral lines ?*

Ans. For shortest wavelength of Paschen series, $n_1 = 3$, $n_2 = \infty$

$$\therefore \quad \frac{1}{\lambda_s} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right] = \frac{R}{9}$$

or

$$\lambda_s = \frac{9}{R} = \frac{9}{1.097 \times 10^7}$$

$$= 8.2041 \times 10^{-7} \text{ m} = \mathbf{8204.1 \text{ Å}}$$

12.4. *A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level ?*

Ans. Here $E = 2.3\,\text{eV} = 2.3 \times 1.6 \times 10^{-19}\,\text{J}$

As $E = h\nu$

∴ Frequency,

$$\nu = \frac{E}{h} = \frac{2.3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

$$= \mathbf{5.6 \times 10^{14}\ Hz.}$$

12.5. *The ground state energy of hydrogen atom is – 13.6 eV. What are the kinetic and potential energies of the electron in his state ?*

Ans. Total energy, $E = -13.6\,\text{eV}$

$$\text{K.E.} = -E = -(-13.6) = \mathbf{13.6\ eV}$$

$$\text{P.E.} = -2\,\text{K.E.} = -2 \times 13.6 = \mathbf{-27.2\ eV.}$$

12.6. *A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of photon.* **[CBSE OD 14C]**

Ans. Energy of an electron in the *n*th orbit of H-atom,

$$E_n = -\frac{13.6}{n^2}\,\text{eV}$$

Energy in the ground ($n = 1$) level,

$$E_1 = -\frac{13.6}{1^2} = -13.6\,\text{eV}$$

Energy in the fourth ($n = 4$) level,

$$E_4 = -\frac{13.6}{4^2} = -0.85\,\text{eV}$$

$$\Delta E = E_4 - E_1$$

$$= -0.85 - (-13.6) = 12.75\,\text{eV}$$

$$= 12.75 \times 1.6 \times 10^{-19}\,\text{J}$$

As $\Delta E = h\nu = \frac{hc}{\lambda}$

∴ Wavelength,

$$\lambda = \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{12.75 \times 1.6 \times 10^{-19}}\,\text{m}$$

$$= 0.975 \times 10^{-7}\,\text{m} = \mathbf{975\ \text{Å}}$$

$$\text{Frequency, } \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{0.975 \times 10^{-7}}$$

$$= \mathbf{3.077 \times 10^{15}\ Hz.}$$

12.7 *(a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the n = 1, 2 and 3 levels. (b) Calculate the orbital period in each of these levels.*

Ans. (*a*) Speed of the electron in Bohr's *n*th orbit is

$$v_n = \frac{2\pi ke^2}{nh} = \frac{v_1}{n}$$

Speed of the electron in Bohr's first ($n = 1$) orbit is

$$v_1 = \frac{2\pi ke^2}{h}$$

$$= \frac{2 \times 3.14 \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.63 \times 10^{-34}}$$

$$= \mathbf{2.186 \times 10^6\ ms^{-1}}$$

$$v_2 = \frac{v_1}{2} = \mathbf{1.093 \times 10^6\ ms^{-1}}$$

$$v_3 = \frac{v_1}{3} = \mathbf{0.729 \times 10^6\ ms^{-1}}.$$

(*b*) Orbital period of electron in Bohr's first orbit is

$$T_1 = \frac{2\pi r_1}{v_1}$$

$$= \frac{2 \times 3.14 \times 0.53 \times 10^{-10}}{2.186 \times 10^6}\,\text{s}$$

$$= \mathbf{1.52 \times 10^{-16}\,s}$$

As $T_n = n^3 T_1$

∴ $T_2 = (2)^3 \times 1.52 \times 10^{-16}$

$= 12.16 \times 10^{-16} = \mathbf{1.22 \times 10^{-15}\,s}$

$T_3 = (3)^3 \times 1.52 \times 10^{-16}$

$= 41.04 \times 10^{-16} = \mathbf{4.10 \times 10^{-15}\ s.}$

12.8. *The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the n = 2 and n = 3 orbits ?* **[CBSE D 14C]**

Ans. Here

$$r_1 = 5.3 \times 10^{-11}\,\text{m}$$

As $r_n = n^2 r_1$

∴ $r_2 = 2^2 \times 5.3 \times 10^{-11} = \mathbf{2.12 \times 10^{-10}\,m}$

$r_3 = 3^2 \times 5.3 \times 10^{-11} = \mathbf{4.77 \times 10^{-10}\,m.}$

12.9. *A 12.75 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted ?*

Ans. Here $\Delta E = 12.75\,\text{eV}$

Energy of an electron in *n*th orbit of hydrogen atom is

$$E_n = -\frac{13.6}{n^2}\,\text{eV}$$

In ground state, $n = 1$,

$$E_1 = -\frac{13.6}{1^2} = -13.6\,\text{eV}$$

Energy of an electron in the excited state after absorbing a photon of 12.75 eV energy becomes

$$E_n = -13.6 + 12.75 = -0.85\,\text{eV}$$

$$\therefore \quad n^2 = -\frac{13.6}{E_n} = -\frac{13.6}{-0.85} = 16 \quad \text{or} \quad n = 4$$

Thus the electron gets excited to $n = 4$ state.

$\therefore$ Total number of wavelengths in emission spectrum

$$= \frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$$

The possible emission lines are shown in Fig. 12.23.

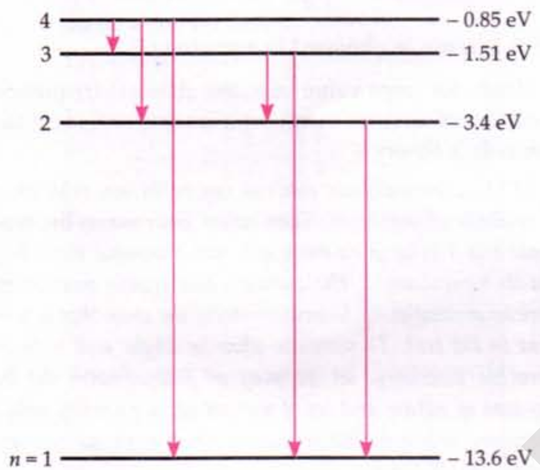

Fig. 12.23

Emitted wavelength,

$$\lambda_{if} = \frac{hc}{E_i - E_f} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{E_i - E_f} = \frac{19.8 \times 10^{-26}}{E_i - E_f} \text{m}$$

$$\lambda_{43} = \frac{19.8 \times 10^{-26}}{(-0.85 + 1.51) \times 1.6 \times 10^{-19}} = \frac{19.8 \times 10^{-7}}{0.66 \times 1.6}$$

$$= 28.409 \times 10^{-7} \text{m} = \mathbf{28409\ Å}$$

$$\lambda_{42} = \frac{19.8 \times 10^{-26}}{(-0.85 + 3.4) \times 1.6 \times 10^{-19}} = \frac{19.8 \times 10^{-7}}{2.55 \times 1.6}$$

$$= 4.8529 \times 10^{-7} \text{m} = \mathbf{4852.9\ Å}$$

$$\lambda_{41} = \frac{19.8 \times 10^{-26}}{(-0.85 + 13.6) \times 1.6 \times 10^{-19}} = \frac{19.8 \times 10^{-7}}{12.75 \times 1.6}$$

$$= 0.9706 \times 10^{-7} \text{m} = \mathbf{970.6\ Å}$$

$$\lambda_{32} = \frac{19.8 \times 10^{-26}}{(-1.51 + 3.4) \times 1.6 \times 10^{-19}} = \frac{19.8 \times 10^{-7}}{1.89 \times 1.6}$$

$$= 6.5476 \times 10^{-7} \text{m} = \mathbf{6547.6\ Å}$$

$$\lambda_{31} = \frac{19.8 \times 10^{-26}}{(-1.51 + 13.6) \times 1.6 \times 10^{-19}} = \frac{19.8 \times 10^{-7}}{12.09 \times 1.6}$$

$$= 1.0236 \times 10^{-7} \text{m} = \mathbf{1023.6\ Å}$$

$$\lambda_{21} = \frac{19.8 \times 10^{-26}}{(-3.4 + 13.6) \times 1.6 \times 10^{-19}} = \frac{19.8 \times 10^{-7}}{10.2 \times 1.6}$$

$$= 1.2132 \times 10^{-7} \text{m} = \mathbf{1213.2\ Å}$$

12.10. *In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 m/s. (Mass of earth = 6.0×10^{24} kg)*

Ans. According to Bohr's quantisation condition of angular momentum,

Angular momentum of the earth around the sun,

$$mvr = \frac{nh}{2\pi}$$

$$\therefore \quad n = \frac{2\pi mvr}{h}$$

$$= \frac{2 \times 3.14 \times 6.0 \times 10^{24} \times 1.5 \times 10^{11} \times 3 \times 10^4}{6.6 \times 10^{-34}}$$

$$= \mathbf{2.57 \times 10^{74}}.$$

12.11. *Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better :*

(a) Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model ?

(b) Is the probability of backward scattering (i.e., scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model ?

(c) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide ?

(d) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil ?

Ans. (*a*) About the same. This is because we are considering the average angle of deflection.

(*b*) Much less, because there is no such massive core (nucleus) in Thomson's model as in Rutherford's model.

(*c*) This suggests that scattering is predominantly due to a single collision, because the chance of a single collision increases linearly with the number of the target atoms, and hence linearly with the thickness of the foil.

(*d*) In Thomson model, positive charge is distributed uniformly in the atom. So single collision causes very little deflection. The observed average scattering angle can be explained only by considering multiple scattering. Hence it is wrong to ignore multiple scattering in Thomson's model.

12.12. *The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulumb attraction by a factor of about 10^{-40}. An alternative way of* *looking at this fact is to estimate the radius of the first Bohr o ut of a hydrogen atom if the electron and proton were bounc by gravitational attraction. You will find the answer interesting.*

Ans. The radius of the first orbit in Bohr's model is given by

$$r_0 = \frac{h^2}{4\pi^2 m_e k e^2}$$

If instead of electrostatic attraction between electron and proton, we consider the atom bound by gravitational force $\frac{Gm_p m_e}{r^2}$, then the term ke^2 should be replaced by $Gm_p m_e$. The radius of the first Bohr orbit in a gravitationally bound hydrogen atom will be

$$r_0^G = \frac{h^2}{4\pi^2 G m_p m_e^2}$$

$$= \frac{(6.6\times10^{-34})^2}{4\times9.87\times6.67\times10^{-11}\times1.6725\times10^{-27}\times(9.1\times10^{-31})^2}$$

$$= 1.194\times10^{29}\text{ m} \approx \mathbf{1.2\times10^{29}\text{ m}}$$

This radius is much greater than the estimated size of the whole universe.

12.13. *Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level n to level (n − 1). For large n, show that this frequency equals the classical frequency of revolution of the electron in the orbit.*

[CBSE Sample Paper 11]

Ans. From Bohr's theory, the frequency ν of the radiation emitted when an electron de-excites from level n_2 to level n_1 is given by

$$\nu = \frac{2\pi^2 mk^2 Z^2 e^4}{h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Given $n_1 = n - 1, \quad n_2 = n$

$$\therefore \quad \nu = \frac{2\pi^2 mk^2 Z^2 e^4}{h^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{2\pi^2 mk^2 Z^2 e^4}{h^3}\left[\frac{n^2-(n-1)^2}{(n-1)^2 n^2}\right]$$

$$= \frac{2\pi^2 mk^2 Z^2 e^4}{h^3}\,\frac{(2n-1)}{(n-1)^2 n^2}$$

For large n, $2n - 1 \simeq 2n$ and $n - 1 \simeq n$ and for hydrogen $Z = 1$

$$\therefore \quad \nu = \frac{2\pi^2 mk^2 e^4}{h^3}\times\frac{2n}{n^2 \cdot n^2} = \frac{4\pi^2 mk^2 e^4}{n^3 h^3} \quad ...(1)$$

Now in Bohr's model,

Velocity of electron in nth orbit $= \frac{nh}{2\pi mr}$

and radius of nth orbit $= \frac{n^2 h^2}{4\pi^2 mk e^2}$

Thus orbital frequency of electron in nth orbit is

$$\nu_c = \frac{v}{2\pi r} = \frac{1}{2\pi r}\times\frac{nh}{2\pi mr}$$

$$= \frac{nh}{4\pi^2 m}\times\left(\frac{4\pi^2 mk e^2}{n^2 h^2}\right)^2$$

$$= \frac{4\pi^2 mk^2 e^4}{n^3 h^3}$$

which is same as obtained in equation (1).

Hence for large value of n, the classical frequency of revolution of electron in nth orbit is same as that obtained from Bohr's theory.

12.14. *Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size ? Why is an atom not, say, thousand times bigger than its typical size ? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom (≈ 10^{-10} m)*

(a) Construct a quantity with the dimensions of length from the fundamental constants e, m_e, and c. Determine its numerical value.

(b) You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, m_e, and e will yield the right atomic size. Construct a quantity with the dimension of length from h, m_e, and e and confirm that its numerical value has indeed the correct order of magnitude.

Ans. (*a*) The quantity is $\frac{ke^2}{mc^2}$ or $\frac{e^2}{4\pi\varepsilon_0 mc^2}$

which has the dimension of length.

$$\left[\frac{e^2}{4\pi\varepsilon_0 mc^2}\right] = \frac{Q^2}{M^{-1}L^{-3}T^2Q^2M\,.(LT^{-1})^2} = L$$

Also,

$$\frac{e^2}{4\pi\varepsilon_0 mc^2} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{9.1\times10^{-31}\times(3\times10^8)^2}\text{ m}$$

$$= \mathbf{2.8\times10^{-15}\text{ m}}$$

This length is much smaller than the typical atomic size (≈ 10^{-10} m).

(b) $\frac{4\pi\varepsilon_0 h^2}{4\pi^2 me^2}$

$$= \frac{(6.6\times10^{-34})^2}{4\times9.87\times9\times10^9\times9.1\times10^{-31}\times(1.6\times10^{-19})^2}\text{ m}$$

$$= 5.26\times10^{-11}\text{ m}$$

$$\simeq \mathbf{0.53\times10^{-10}\ m\ or\ 0.53\ \text{Å}}$$

The length is of the order of atomic size ($\sim 10^{-10}$ m).

12.15. *The total energy of an electron in the first excited state of the hydrogen atom is about − 3.4 eV.*

(a) What is the kinetic energy of the electron in this state ?

(b) What is the potential energy of the electron in this state ?

(c) Which of the answers above would change if the choice of the zero of potential energy is changed ? **[CBSE D14C]**

Ans. K.E. of an electron in *n*th orbit,

$$T = \frac{1}{2}\frac{kZe^2}{r^2}$$

P.E. of an electron in *n*th orbit,

$$V = -\frac{kZe^2}{r} = -2T$$

Total energy,

$$E = T + V = T - 2T = -T$$

(a) Kinetic energy,

$$T = -E = -(-3.4) = \mathbf{3.4\ eV.}$$

(b) Potential energy,

$$V = -2T = -2\times3.4 = \mathbf{-6.8\ eV.}$$

(c) If the zero of the potential energy is chosen differently, kinetic energy does not change. Potential energy and hence total energy will be affected.

12.16. *If Bohr's quantisation postulate (angular momentum = nh/2π) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun ?*

Ans. Angular momenta associated with planetary motion are incomparably large relative to $h/2\pi$. For example, angular momentum of the earth in its orbital motion is of the order to $10^{70}\ h/2\pi$. In terms of the Bohr's quantisation postulate, this corresponds to a very large value of n (of the order of 10^{70}). For such large values of *n*, the differences in the successive energies and angular momenta of the quantised levels of the Bohr model are so small compared to the energies and angular momenta respectively of the levels that one can, for all practical purposes, consider the levels continuous.

12.17. *Obtain the first Bohr's radius and the ground state energy of a 'muonic hydrogen atom' (i.e., an atom in which a negatively charged muon (μ^-) of mass about 207 m_e orbits around a proton).*

Ans. In Bohr's model, the radius of *n*th orbit is

$$r = \frac{n^2h^2}{4\pi^2\ mk\ Ze^2} \quad i.e., \quad r \propto \frac{1}{m}$$

Now in a muonic hydrogen atom, a negatively charged muon (μ^-) of mass 207 m_e revolves around a proton.

Therefore, we can write

$$\frac{r_\mu}{r_e} = \frac{m_e}{m_\mu} = \frac{m_e}{207\ m_e}$$

$$\therefore \quad r_\mu = \frac{1}{207}\times r_e = \frac{1}{207}\times0.53\times10^{-10}\text{ m}$$

$$= \mathbf{2.5\times10^{-13}\ m}$$

Energy of electron in *n*th orbit,

$$E = -\frac{2\pi^2\ mk^2\ Z^2\ e^4}{n^2h^2}$$

When all other factors are fixed, $E \propto m$

$$\therefore \quad \frac{E_\mu}{E_e} = \frac{m_\mu}{m_e} = \frac{207\ m_e}{m_e}$$

or $$E_\mu = 207\ E_e = -207\times13.6\text{ eV}$$

$$\simeq \mathbf{-2.8\ keV.}$$

Text Based Exercises

TYPE A : VERY SHORT ANSWER QUESTIONS (1 mark each)

1. Name the first atom model.
2. Which model of the atom assumes nearly a continuous distribution of mass in it ?
3. What are alpha particles ?
4. The size of the nucleus can be estimated by the scattering of which particles ?
5. Name the experiment responsible for the discovery of atomic nucleus.

6. What was the main conclusion of Rutherford's experiment on the scattering of alpha particles by thin foils ? [CBSE D 96]
7. Define distance of closest approach. [Haryana 96]
8. Define impact parameter. [Haryana 07, 09 ; Punjab 01]
9. On what factors does the shape of trajectory of scattered α-particles depend ?
10. How is the impact parameter b related to the scattering angle θ ? [Haryana 92]
11. What is the angle of scattering for zero impact parameter ?
12. For a projectile of given energy, will the scattering angle increase or decrease with increase in impact parameter ?
13. Why did Rutherford experiment require nuclear model of the atom ?
14. Most of the mass of an atom is with the positive charge. In case of hydrogen atom, what fraction of the atomic mass is with the positive charge ?
15. What is the Bohr quantisation condition of the angular momentum of an electron in the second orbit ? [CBSE D 94C]
16. What is the angular momentum of an electron in the 3rd orbit of an atom ? [CBSE F 93]
17. State Bohr's postulate of hydrogen atom which successfully explains the emission lines in the spectrum of hydrogen atom. [CBSE OD 15]
18. What is the order of the speed of electron in a hydrogen atom in ground state ? [Punjab 09C]
19. The ground state energy of hydrogen atom is -13.6 eV. What are the kinetic and potential energies of electron in this state ? [CBSE OD 10, 11]
20. The total electrical energy of an electron in the first excited state of hydrogen atom is about -3.4 eV. What is the kinetic energy of the electron in this state ? [CBSE OD 95]
21. The total electrical energy of an electron in the first excited state of hydrogen atom is about -3.4 eV. What is the potential energy of the electron in this state ? [CBSE D 95]
22. What is the potential energy of an electron when it is far away from the nucleus ?
23. Write the expression for Bohr's radius in hydrogen atom. [CBSE D 10]
24. How much is the radius of Bohr's innermost orbit ?
25. What is the ratio of radii of the orbits corresponding to first excited state and ground state in a hydrogen atom ? [CBSE D 10]
26. The radius of innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What is the radius of orbit in the second excited state ? [CBSE D 10]
27. What is fine structure constant ? Give its value. [Punjab 09 C]
28. What is Rydberg's constant ? Give its value.
29. Define ionisation energy. What is its value for a hydrogen atom ? [CBSE OD 11]
30. The ionisation potential of an atom is 24.6 V. How much is its ionisation energy ?
31. Write an empirical relation for Paschen series lines of hydrogen spectrum. [Haryana 95]
32. Name the series of hydrogen spectrum lying in the ultraviolet region.
33. Name the series of hydrogen spectrum which has least wavelength. [CBSE D 95C]
34. Out of three radiations of wavelengths 8000 Å, 5000 Å and 1000 Å, which one corresponds to Lyman series of hydrogen spectrum ? [CBSE F 94]
35. Name the spectral series of hydrogen spectrum lying in the infrared region. [Punjab 09 C]
36. Name the spectral series of hydrogen spectrum which lies in the visible region of the e.m. spectrum. [CBSE D 96C]
37. In which series of hydrogen spectrum, the transitions involve the largest changes of energy ?
38. What do you mean by short wavelength limit of a series ?
39. When is H_α line of the Balmer series in the emission spectrum of hydrogen atom obtained ? [CBSE D13 C]

Answers

1. Thomson model of the atom.
2. Thomson model of the atom.
3. Alpha-particles are doubly ionised helium atoms *i.e.* helium nuclei which have mass number 4 and charge number 2.
4. Alpha particles.
5. Rutherford's α- particle scattering experiment.
6. Rutherford concluded that there is a central massive positively charged core, called nucleus, inside every atom.

7. The distance of closest approach is the distance between the centre of the nucleus and the point from which an α-particle approaching directly to the nucleus returns.
8. Impact parameter is the perpendicular distance of the velocity vector of the α-particle from the central line of the nucleus, when the particle is far away from the atom.
9. The shape of the trajectory of scattered α-particles depends on (*i*) impact parameter and (*ii*) nature of the potential field encountered by the α-particle.
10. $b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot \frac{\theta}{2}}{E}$

$$= \frac{1}{4\pi\varepsilon_0} \frac{Ze^2 \cot \frac{\theta}{2}}{\frac{1}{2} mv^2}$$

11. 180°
12. The scattering angle will decrease with the increase in impact parameter.
13. Nuclear model was needed to explain large angle scattering of α-particles.
14. A hydrogen atom contains one proton (+ ve charge) and one electron (– ve charge). As the mass of a proton is 1836 times that of an electron, so 1836/1837 part of the atomic mass is associated with the positive charge.
15. $L = 2 \cdot \frac{h}{2\pi} = \frac{h}{\pi}$.
16. $L = 3 \cdot \frac{h}{2\pi}$

$$= \frac{3 \times 6.6 \times 10^{-34} \times 7}{2 \times 22}$$

$$= 3.15 \times 10^{-34} \text{ Js.}$$

17. An atom emits radiation in the form of discrete energy photons only when an electron jumps from a higher to a lower orbit and the energy of the photon is equal to the difference in energy in the two orbits. Thus, $h\nu = E_2 - E_1$.
18. 10^6 ms^{-1}.
19. Total energy,

$$E = -13.6 \text{ eV}$$

Kinetic energy,

$$T = -E = +\mathbf{13.6 \text{ eV}}$$

Potential energy,

$$V = -2T = -2 \times 13.6$$

$$= -\mathbf{27.2 \text{ eV}}.$$

20. K.E. of the electron = – Total energy of the electron $= \mathbf{+3.4 \text{ eV}}$.
21. Potential energy = 2 × Total energy

$$= 2 \times (-3.4) = -\mathbf{6.8 \text{ eV}}.$$

22. Zero.
23. Bohr's radius, $r_0 = \frac{h^2}{4\pi^2 mke^2}$.
24. $r = 0.53$ Å.
25. $\frac{\text{Radius of orbit of first excited state } (n=2)}{\text{Radius of orbit of ground state } (n=1)}$

$$= \frac{2^2}{1^2} = \mathbf{4:1}$$

26. For second excited state, $n = 3$

Therefore, $r_3 = (3)^2 r_0 = 9 \times 5.3 \times 10^{-11}$

$$= \mathbf{4.72 \times 10^{-10} \text{ m}}.$$

27. Fine structure constant is a dimensionless constant defined by

$$\alpha = \frac{2\pi ke^2}{ch}$$

Value of $\alpha = \frac{1}{137}$

28. Rydberg's constant R is defined by

$$R = \frac{2\pi^2 mk^2 e^4}{ch^3}$$

Value of $R = 1.09479 \times 10^7 \text{ m}^{-1}$.

29. Ionisation energy is defined as the energy required to knock an electron completely out of the atom.

Ionisation energy of hydrogen

$$= E_\infty - E_1 = 0 - (-13.6) = \mathbf{13.6 \text{ eV}}.$$

30. Ionisation energy = 24.6 eV.
31. Paschen series is defined by the wave number,

$$\bar{\nu} = \frac{1}{R}\left[\frac{1}{3^2} - \frac{1}{n^2}\right], \text{ where } n = 4, 5, 6...$$

32. Lyman series.
33. Lyman series
34. 1000 Å, because Lyman series lies in the ultraviolet region of the spectrum.
35. Paschen, Bracket and Pfund series lie in the infrared region.
36. Balmer series lies in the visible region.
37. Lyman series.
38. The wavelength corresponding to the most energetic transition in a series gives the short wavelength limit of the series.
39. When electron jumps from $n_2 = 3$ level to $n_1 = 2$ level.

TYPE B : SHORT ANSWER QUESTIONS (2 or 3 marks each)

1. Describe Thomson's model of an atom. Why was this model discarded later on ?
2. Draw a labelled diagram of experimental set up of Rutherford's alpha particle scattering experiment. Write two important inferences drawn from this experiment. [CBSE OD 05C]
3. (*a*) In an experiment on α-particle scattering by a thin foil, draw a plot showing the number of particles scattered versus the scattering angle θ.
   (*b*) Why is it that a very small fraction of the particles is scattered at $\theta > 90°$?
   (*c*) Write two important conclusions that can be drawn regarding the structure of the atom from the study of this experiment. [CBSE F 13]
4. In Rutherford scattering experiment, draw the trajectory traced by α-particles in the Coulomb field of target nucleus and explain how this led to estimate the size of the nucleus. [CBSE OD 15C]
5. Describe Rutherford's model of the atom. Give its limitations. [Himachal 03, 04]
6. Draw a schematic arrangement of the Geiger-Marsden experiment. How did the scattering of α-particles by a thin foil of gold provide an important way to determine an upper limit on the size of the nucleus ? Explain briefly. [CBSE F 2008 ; OD 2009]
7. What is impact parameter. How does it influence the trajectory of an α –particle scattered by heavy nucleus ? What is the value of impact parameter for a head on collision ?
8. Obtain Bohr's quantisation condition of angular momentum on the basis of wave picture of an electron. [CBSE OD 90 ; Punjab 92]
9. Show that Bohr's second postulate, 'the electron revolves around the nucleus only in certain fixed orbits without radiating energy' can be explained on the basis of de Broglie hypothesis of wave nature of electron. [CBSE OD 08, 11]
10. Show that the radius of the orbit in hydrogen atom varies as n^2, where n is the principal quantum number of the atom. [CBSE D 15]
11. Calculate the speed of electron revolving around the nucleus of a hydrogen atom in order that it may not be pulled into the nucleus by electrostatic attraction. [CBSE D 1992]

    Or

    Show that the speed of an electron in the innermost orbit of H-atom is 1/ 137 times the speed of light in vacuum.
12. State Bohr's postulate for the 'permitted orbits' for the electron in a hydrogen atom.

    Use this postulate to prove that the circumference of the nth permitted orbit for the electron can 'contain' exactly n wavelengths of the de-Broglie wavelength associated with the electron in that orbit. [CBSE Sample Paper 08]
13. Using Bohr's postulates, obtain the expressions for (*i*) kinetic energy and (*ii*) potential energy of the electron in stationary state of hydrogen atom.

    Draw the energy level diagram showing how the transitions between energy levels result in the appearance of Lymann Series. [CBSE D 13]
14. Using Bohr's postulates, obtain the expression for the total energy of the electron in the stationary states of the hydrogen atom. Hence draw the energy level diagram showing how the line spectra corresponding to Balmer series occur due to transition between energy levels. [CBSE D 13]
15. (*a*) Using Bohr's postulates, obtain the expression for total energy of the electron in the nth orbit of hydrogen atom.
    (*b*) What is the significance of negative sign in the expression for the energy ?
    (*c*) Draw the energy level diagram showing how the line spectra corresponding to Paschen series occur due to transition between energy levels. [CBSE D 13]
16. Given the ground state energy $E_0 = -13.6\,\text{eV}$ and Bohr radius $a_0 = 0.53$ Å. Find out how de-Broglie wavelength associated with the electron orbiting in the ground state would change when it jumps into the first excited state. [CBSE OD 15]
17. When an electron in hydrogen atom jumps from the third excited state to the ground state, how would the de Broglie wavelength associated with the electron change ? Justify your answer. [CBSE OD 15]

Answers

1. Refer answer to Q.1 on page 12.1.
2. See Fig. 12.2 on page 12.1.
   Two important inferences drawn from the experiment are
   (*i*) The most of the mass and the entire positive charge of the atom is concentrated in a very small volume of the atom called nucleus.
   (*ii*) The nuclear radius is about 1/10,000 of the atomic radius.
3. (*a*) See Fig. 12.3 on page 12.2.
   (*b*) The size of the nucleus is very small, about 1/10000 of the atomic size. So a very small fraction of the incident α-particles is scattered at $\theta > 90°$ by the atomic nucleus.
   (*c*) Refer answer to the above question.
4. See Fig. 12.4 and refer answer to Q. 3 on page 12.2.
5. Refer answer to Q. 5 on page 12.4.
6. See Fig. 12.2. Refer answer to Q. 3 on page 12.2.
7. Refer answer to Q. 4 on page 12.3.
8. Refer answer to Q. 6 on page 12.6.
9. Refer answer to Q. 6 on page 12.6.
10. Refer to the solution of Problem 6 on page 12.22.
11. Refer answer to Q. 8 on page 12.7.
12. Refer to the solution of Problem 7(*a*) on page 12.22.
13. Refer answer to Q. 8 on page 12.7 and see Fig. 12.12.
14. Refer answer to Q. 8 on page 12.7 and see Fig. 12.12.
15. Refer answer to Q. 8 on page 12.7 and see Fig. 12.12.
16. de-Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{E_2}{E_1}}$$

As $E_n \propto \frac{1}{n^2}$, so for $n = 2$, $E_2 = \frac{E_1}{2^2} = \frac{E_1}{4}$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\Rightarrow \quad \lambda_2 = 2\lambda_1.$$

17. $E_4 = \frac{E_1}{4^2} = \frac{E_1}{16}$

$$\Rightarrow \quad \frac{E_4}{E_1} = \frac{1}{16}$$

$$\therefore \quad \frac{\lambda_4}{\lambda_1} = \sqrt{\frac{E_1}{E_4}} = \sqrt{\frac{16}{1}} = 4$$

$$\Rightarrow \quad \lambda_4 = 4\lambda_1.$$

TYPE C : LONG ANSWER QUESTIONS (5 marks each)

1. Draw a labelled diagram for α-particle scattering experiment. Give Rutherford's observations and discuss the significance of this experiment. Obtain the expression which helps us to get an idea of the size of the nucleus, using these observations. [CBSE F 03]
2. (*a*) Draw a schematic arrangement of Geiger-Marsden experiment showing the scattering of α-particles by a thin foil of gold. Why is it that most of the α-particles go straight through the foil and only a small fraction gets scattered at large angles ?
   Draw the trajectory of the α-particle in the Coulomb field of a nucleus. What is the significance of impact parameter and what information can be obtained regarding the size of the nucleus ?
   (*b*) Estimate the distance of closest approach to the nucleus (Z = 80) if a 7.7 MeV α-particle before it comes momentarily to rest and reverses its direction. [CBSE D 15C]
3. (*a*) Write two important limitations of Rutherford model which could not explain the observed features of atomic spectra. How were these explained in Bohr's model of hydrogen atom ?
   Use the Rydberg formula to calculate the wavelength of the H_α line of the Balmer series. (Take $R = 1.03 \times 10^7 \text{ m}^{-1}$).
   (*b*) Using Bohr's postulates, obtain the expression for the radius of the *n*th orbit in hydrogen atom. [CBSE D 15C]
4. State Bohr's postulates. Using these postulates derive an expression for the total energy of an electron in the *n*th orbit of an atom. What does negative value of this energy signify ? What is Bohr's radius ?
   [Haryana 97 ; CBSE OD 97]
5. What is the energy level diagram for an atom ? Calculate the energies of the various energy levels of a hydrogen atom and draw an energy level diagram for it.

6. Using Bohr's postulates, derive the expression for the frequency of radiation emitted when electron in hydrogen atom undergoes transition from higher energy state (quantum number n_i) to the lower state, (n_f).

When electron in hydrogen atom jumps from energy state $n_i = 4$ to $n_f = 3, 2, 1$, identify the spectral series to which the emission lines belong. [CBSE OD 13]

Answers

1. Refer answer to Q. 2 on page 12.1 and Q. 3 on page 12.2.

2. (*a*) See Fig. 12.2 on page 12.2. For most of the α-particles, the impact parameter is large, hence they suffer very small repulsion due to the nucleus and go straight the foil.

   See Fig. 12.4 on page 12.2. The impact parameter decides the shape of the trajectory of an α-particle scattered from a heavy nucleus. Also, it gives an estimate of the size of the nucleus.

   (*b*) Refer to the solution of Example 2 on page 12.4.

3. (*a*) According to Rutherford's atomic model,

   (*i*) An electron revolving around the nucleus is continuously accelerated. So it continuously loses energy and finally spiral into the nucleus.

   (*ii*) It must emit a continuous spectrum.

   According to Bohr's model of hydrogen atom,

   (*i*) While revolving in a permissible orbit, an electron does not radiate energy.

   (*ii*) Energy is released/absorbed only, when an electron jumps from one stable orbit to another. This results in a discrete spectrum.

   Numerical. For H_α line of Balmer series : $n_i = 2$, $n_f = 3$

   $$\therefore \quad \frac{1}{\lambda_\alpha} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}R$$

   $$\text{or} \quad \lambda_\alpha = \frac{36}{5R} = \frac{36}{5 \times 1.03 \times 10^7}\text{ m}$$

   $$= \frac{36 \times 10^4}{5 \times 103}\text{ nm}$$

   $$= 699.03 \text{ nm} \simeq \mathbf{699 \text{ nm}}.$$

   (*b*) Refer to the solution of Problem 6 on page 12.26.

4. Refer answer to Q. 7 and Q. 8 on page 12.7.

5. Refer answer to Q. 10 on page 12.9.

6. Refer answer to Q. 8 on page 12.7 and Q. 9 on page 12.8.

   $n_i = 4 \rightarrow n_f = 3 \quad \Rightarrow$ Paschen series

   $n_i = 4 \rightarrow n_f = 2 \quad \Rightarrow$ Balmer series

   $n_i = 4 \rightarrow n_f = 1 \quad \Rightarrow$ Lyman series

COMPETITION SECTIONS

Atoms

GLIMPSES

1. **Atom as a constituent of every element.** Every element has characteristic atoms. Atoms of different elements contain electrons, which are completely identical. Atom as a whole is electrically neutral and therefore contains equal amount of positive and negative charges.

2. **Thomson's model of an atom.** An atom consists of a sphere of positively charged matter in which the negatively charged electrons are uniformly embedded like plums in a pudding. This model could not explain scattering of α-particles through thin foils and hence discarded.

3. **Rutherford's model of an atom.** Geiger and Marsden performed experiments on scattering of α-particles from metal foils. A collimated beam of 5.5 MeV of α-particles was allowed to fall on a 2.1×10^{-7} m thin gold foil. The scattered α-particles produced scintillations on a ZnS screen, which were counted at different angles (θ) from the direction of the beam. It was found that most of the α-particles passed undeviated through thin foils but some of them were scattered through very large angles. From the results of these experiments, Rutherford proposed the following model of an atom.

   (*i*) An atom consists of a small and massive central core in which the entire positive charge and almost the whole mass of the atom are concentrated. This core is called the **nucleus**.

   (*ii*) The nucleus occupies a very small space as compared to the size of the atom.

   (*iii*) The atom is surrounded by a suitable number of electrons so that their total negative charge is equal to the total positive charge on the nucleus and the atom as a whole is electrically neutral.

   (*iv*) The electrons revolve around the nucleus in various orbits just as planets revolve around the sun. The centripetal force required for their revolution is provided by the electrostatic attraction between the electrons and the nucleus.

   This model could not explain the stability of the atom because according to classical electromagnetic theory the electron revolving around the nucleus must continuously radiate energy in the form of electromagnetic radiations and hence it should fall into the nucleus.

4. **Distance of closest approach.** When an α-particle of mass m and velocity v moves directly towards a nucleus of atomic number Z, its distance of closest approach is given by

$$r_0 = \frac{2k\, Ze^2}{E} = \frac{4k\, Ze^2}{mv^2}$$

where $E = \frac{1}{2} mv^2$

and $k = \frac{1}{4\pi\, \varepsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$.

5. **Impact parameter.** It is defined as the perpendicular distance of the velocity vector of the α-particle from the centre of the nucleus, when it is far away from the atom. The shape of the trajectory of the scattered α-particle depends on the impact parameter b and the nature of the

potential field. Rutherford deduced the following relationship between the impact parameter b and the scattering angle θ :

$$b=\frac{1}{4\pi\,\varepsilon_0}\cdot\frac{Ze^2\cot\frac{\theta}{2}}{E}=\frac{1}{4\pi\,\varepsilon_0}\cdot\frac{Ze^2\cot\frac{\theta}{2}}{\frac{1}{2}mv^2}.$$

6. **Quantisation or discretisation.** The quantisation or discretisation of a physical quantity means that it cannot vary continuously to have any arbitrary value but can change only discontinuously to take certain specific values. The energy of electrons in an atom is quantised.

7. **Bohr's atom model.** This model is also called *planetary model* of an atom and is based on following postulates :

   (*i*) ***Nuclear concept.*** An atom consists of a small massive central core called nucleus around which planetary electrons revolve. The centripetal force required for their revolution is provided by the electrostatic attraction between the electrons and the nucleus.

   (*ii*) ***Quantum condition.*** Of all the possible circular orbits allowed by the classical theory, the electrons are permitted to circulate only in such orbits in which the angular momentum of an electron is an integral multiple of $h/2\pi$, h being Planck's constant.

   $$L=mvr=\frac{nh}{2\pi},\ n=1,2,3,.....$$

   where n is called *principal quantum number.*

   (*iii*) ***Stationary orbits.*** While revolving in the permissible orbits, an electron does not radiate energy. These non-radiating orbits are called stationary orbits.

   (*iv*) ***Frequency condition.*** An atom can emit or absorb radiation in the form of discrete energy photons only when an electron jumps from a higher to a lower orbit or from a lower to a higher orbit. If E_1 and E_2 are the energies associated with these permitted orbits, then the frequency ν of the emitted or absorbed radiation is given by

   $$h\nu=E_2-E_1$$

8. **Bohr's theory of hydrogen atom.** An electron having charge $-e$ revolves with speed v in a circular orbit of radius r round the nucleus having charge $+e$.

   For a circular orbit,

   $$\frac{mv^2}{r}=\frac{k\,Ze^2}{r}$$

   From quantisation of angular momentum,

   $$L=mvr=\frac{nh}{2\pi}$$

   On solving the above two equations, we get

   Radius of nth orbit,

   $$r=\frac{n^2h^2}{4\pi^2\,mk\,Ze^2}$$

   Speed of electron in nth orbit,

   $$v=\frac{2\pi\,ke^2}{nh}=\alpha\frac{c}{n}=\frac{1}{137}\cdot\frac{c}{n}.$$

   where $\alpha=\dfrac{2\pi\,ke^2}{ch}$ is *fine structure constant.*

   Total energy of an electron in nth orbit is

   $$E_n=\text{K.E.}+\text{P.E.}=\frac{k\,Ze^2}{2r}-\frac{kZe^2}{r}=-\frac{kZe^2}{2r}$$

   or
   $$E_n=-\frac{2\pi^2\,mk^2\,Z^2\,e^4}{n^2h^2}$$

   $$=-\frac{Z^2Rhc}{n^2}=-\frac{13.6\,Z^2}{n^2}\text{ eV}$$

   where
   $$R=\frac{2\pi^2\,mk^2\,e^4}{ch^3}$$

   $$=1.0973\times10^7\ \text{m}^{-1},$$

   is the *Rydberg's constant.*

9. **Spectral series of hydrogen atom.** Whenever an electron makes a transition from a higher energy level n_2 to a lower energy level n_1, the difference of energy appears in the form of a photon of frequency ν given by

   $$\nu=\frac{2\pi^2\,mk^2\,Z^2\,e^4}{h^3}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

   Wave number,

   $$\bar{\nu}=\frac{1}{\lambda}=\frac{\nu}{c}=\frac{2\pi^2\,mk^2\,Z^2\,e^4}{ch^3}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

   or
   $$\bar{\nu}=\frac{1}{\lambda}=RZ^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

Different spectral series for hydrogen atom are as follows :

(*i*) ***Lyman Series.*** Here $n_2 = 2, 3, 4, \ldots$ and $n_1 = 1$. This series lies in the *ultraviolet region*.

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right], \quad n_2 = 2, 3, 4, \ldots$$

(*ii*) ***Balmer Series.*** Here $n_2 = 3, 4, 5, \ldots$ and $n_1 = 2$. This series lies in the *visible region*.

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right], \quad n_2 = 3, 4, 5, \ldots$$

(*iii*) ***Paschen Series.*** Here $n_2 = 4, 5, 6, \ldots$ and $n_1 = 3$. This series lies in the *infrared region*.

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right], \quad n_2 = 4, 5, 6, \ldots$$

(*iv*) ***Brackett Series.*** Here $n_2 = 5, 6, 7, \ldots$ and $n_1 = 4$. This series lies in the *infrared region*.

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right], \quad n_2 = 5, 6, 7, \ldots$$

(*v*) ***Pfund Series.*** Here $n_2 = 6, 7, 8, \ldots$ and $n_1 = 5$. This series lies in the *infrared region*.

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right], \quad n_2 = 6, 7, 8, \ldots$$

10. Energy level diagram. It is a diagram in which the energies of the different stationary states of an atom are represented by parallel horizontal lines, drawn according to some suitable energy scale.

11. Failure of Bohr's model. This model is applicable only to hydrogen-like atoms and fails in case of higher atoms. It could not explain the fine structure of the spectral lines in the spectrum of hydrogen atom.

12. Excitation energy. It is defined as the energy required by an electron of an atom to jump from its ground state to any one of its excited states.

13. Ionisation energy. It is defined as the energy required to remove an electron from an atom, *i.e.*, the energy required to take an electron from its ground state to the outermost orbit ($n = \infty$).

14. Excitation potential. It is that accelerating potential which gives to a bombarding electron sufficient energy to excite the target atom by raising one of its electrons from an inner to an outer orbit.

15. Ionisation Potential. It is that accelerating potential which gives to a bombarding electron sufficient energy to ionise the target atom by knocking one of its electrons completely out of the atom.